NPS-IJWA-01-010

HETEROGENEOUS SALVO MODEL FOR THE NAVY AFTER NEXT

Michael D. Johns
Steven E. Pilnick
Wayne P. Hughes, Jr.

January 2001

The Institute for Joint Warfare Analysis
Naval Postgraduate School
Monterey, California

NPS-IJWA-01-010

HETEROGENEOUS SALVO MODEL FOR THE NAVY AFTER NEXT

Michael D. Johns
Steven E. Pilnick
Wayne P. Hughes, Jr.

January 2001

Institute for Joint Warfare Analysis
Naval Postgraduate School
Monterey, California

RADM David R. Ellison, USN
Superintendent

Richard Elster
Provost

This report was prepared for and partially funded by:

Institute for Joint Warfare Analysis, Naval Postgraduate School

This report was prepared by:

Institute for Joint Warfare Analysis, Naval Postgraduate School

Authors:

Michael D. Johns
Steven E. Pilnick
Wayne P. Hughes, Jr.

Reviewed by:

GORDON SCHACHER
Director
Institute for Joint Warfare Analysis

Released by:

DAVID W. NETZER
Associate Provost and
Dean of Research

REPORT DOCUMENTATION PAGE

Form Approved OMB No. 0704-0188

1. AGENCY USE ONLY *(Leave blank)*	2. REPORT DATE	3. REPORT TYPE AND DATES COVERED
	January 2001	Technical Report

4. TITLE AND SUBTITLE
Heterogeneous Salvo Model for the Navy After Next

5. FUNDING NUMBERS

6. AUTHOR(S)
Johns, Michael D.; Pilnick, Steven E., & Hughes, Wayne P.

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Naval Postgraduate School
Monterey, CA 93943-5000

8. PERFORMING ORGANIZATION REPORT NUMBER
NPS-IJWA-01-010

9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES)

10. SPONSORING / MONITORING AGENCY REPORT NUMBER

11. SUPPLEMENTARY NOTES
The views expressed in this thesis are those of the author and do not reflect the official policy or position of the Department of Defense or the U.S. Government.

12a. DISTRIBUTION / AVAILABILITY STATEMENT
Approved for public release; distribution is unlimited.

12b. DISTRIBUTION CODE

13. ABSTRACT *(maximum 200 words)*
The Navy Warfare Development Command has taken the lead in studying needed Capabilities for the Navy After Next. Amongst the ideas they are considering are innovative special purpose littoral warfare platforms as well as alternative relationships between platforms, sensors, weapons, and information. This thesis presents a low-resolution model for analysis of Navy After Next concepts and demonstrates the potential use of the model. Presented is an adaptation of the existing Hughes Salvo Model which had been limited to analysis of engagements between forces composed of identical units, i.e., homogeneous forces. This heterogeneous extension is an analytical device that captures the unique combat characteristics of individual units. The model helps decision makers understand salvo warfare of heterogeneous forces by simplifying the complex relationships within and between forces during battle. Using a previous work that tested Hughes' model, the accuracy of this heterogeneous salvo model is examined by comparing results. This thesis further demonstrates the strength of the heterogeneous salvo model through an analysis of a hypothetical campaign scenario and through an examination of alternative tactics.

14. SUBJECT TERMS
Combat Models, Salvo Model, Hughes Salvo Model, Naval Tactics, Campaign Analysis

15. NUMBER OF PAGES
84

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFIACTION OF ABSTRACT	20. LIMITATION OF ABSTRACT
Unclassified	Unclassified	Unclassified	UL

NSN 7540-01-280-5500

Standard Form 298 (Rev. 2-89)
Prescribed by ANSI Std. 239-18

THIS PAGE INTENTIONALLY LEFT BLANK

ABSTRACT

The Navy Warfare Development Command has taken the lead in studying needed Capabilities for the Navy After Next. Amongst the ideas they are considering are innovative special purpose littoral warfare platforms as well as alternative relationships between platforms, sensors, weapons, and information. This thesis presents a low-resolution model for analysis of Navy After Next concepts and demonstrates the potential use of the model. Presented is an adaptation of the existing Hughes Salvo Model which had been limited to analysis of engagements between forces composed of identical units, i.e., homogeneous forces. This heterogeneous extension is an analytical device that captures the unique combat characteristics of individual units. The model helps decision makers understand salvo warfare of heterogeneous forces by simplifying the complex relationships within and between forces during battle. Using a previous work that tested Hughes' model, the accuracy of this heterogeneous salvo model is examined by comparing results. This thesis further demonstrates the strength of the heterogeneous salvo model through an analysis of a hypothetical campaign scenario and through an examination of alternative tactics.

THIS PAGE INTENTIONALLY LEFT BLANK

TABLE OF CONTENTS

LIST OF FIGURES

THIS PAGE INTENTIONALLY LEFT BLANK

LIST OF TABLES

THIS PAGE INTENTIONALLY LEFT BLANK

I. INTRODUCTION

A. BACKGROUND

The Navy Warfare Development Command (NWDC), in coordination with the Defense Advanced Research Projects Agency (DARPA), is studying efforts to identify needed Capabilities for the Navy After Next (CNAN). The CNAN project examines those leading technologies, capabilities, and doctrine that will best contribute to the United States Navy twenty years in the future. CNAN is unique from other future planning organizations in that the foundations of its work are not limited to the current feasibility of the ideas. Instead of designing the Navy after Next using current trends, CNAN attempts to step "outside the box" and examine these ideas and the steps necessary to achieve those goals.

Joint Vision 2020 [Ref.1] and *Forward...From the Sea, The Navy Operational Concept* [Ref.2] provide Navy leaders with insight into how future military threats will manifest themselves and how U.S. military forces will work together to fight against these threats. *Joint Vision 2020* also stresses the importance of our ability to recognize emerging technologies and successfully incorporate them into the military organization. The ultimate goal is to increase the capabilities of U.S. forces. Failure to do so, coupled with the enemy successes, increases the chance of failure in future conflicts.

One future capability under examination defines an alternative relationship between platforms, sensors, weapons, and information that enables a "powerful, fast striking geographically dispersed force that exploits information superiority to rapidly overwhelm its adversaries [Ref. 3]." The result is network-centric operations (NCO), an innovative concept that shifts operational focus from individual nodes, or platforms, to a network of nodes. NCO allows a force to synchronously distribute its assets throughout a battle space while constantly adapting to changes in its environment.

A second possibility for CNAN is the Streetfighter concept. The Streetfighter characteristics are only loosely defined but generally thought of as small, high-speed, affordable, and perhaps expendable surface combatants [Ref. 4]. It is evident that Streetfighter and NCO are complementary features of a CNAN force. For motivation and

in an analysis example, this thesis will use a generalized version of a future CNAN design coupling NCO and Streetfighters. To assist in analysis, the thesis uses mathematical terminology to describe the CNAN force in its most general form. The Navy After Next will consist of a network of platforms, sensors, and weapons, integrated in a way to provide military decision makers with superior battlespace awareness. Each network node represents a military unit, whether a platform, sensor, or weapon, with some characteristic parameters for offensive and defensive capabilities. The arcs between nodes represent the multi-dimensional links between units. These arcs may represent communication links, data transfer, or some other means of synchronous linkage. Figure 1 shows a three-dimensional network representation of the different components that might make up the CNAN force. Each different shade represents a unit function, such as a sensor, weapon, or platform, and within each shaded group there are multiple instances of that unit. For example, Figure 1 can represent a CNAN force of four black surface ships, five gray radar systems, and six white weapon systems linked together. The arcs between units may be a type of super-high frequency or line-of-sight radio frequency, while the inter-unit arcs can be satellite communications or future advanced data links. The reader should not infer that the functional layers are independent sub-networks. Rather, as Figure 2 depicts, all nodes are members of one network.

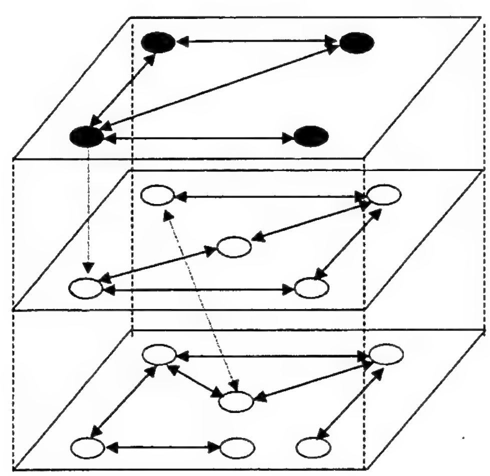

Figure 1. Network Components for the CNAN Force

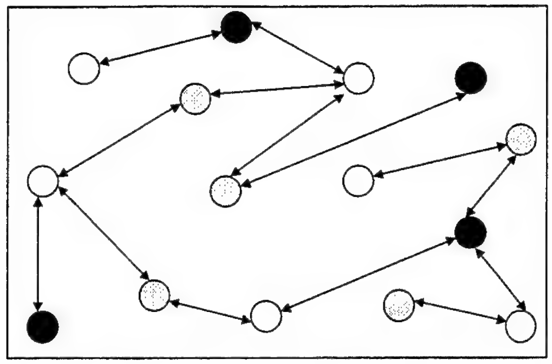

Figure 2. A Two-Dimensional View of a CNAN Force

The greatest advantage that a network-centric force provides is its ability to generate accurate and timely battlespace information while simultaneously disseminating that information throughout the force. This increase in accurate information, in turn, may empower the force as never before by providing a more complete assessment of both friendly and enemy operations. There is also expected to be the additional benefit of a decrease in response time for time-sensitive evolutions such as over-the-horizon targeting in force-on-force engagements.

A sports example to understand CNAN forces is that of a good soccer team. The team is composed of eleven individual players each of whom have particular capabilities (strength, endurance, coordination, and dexterity). The team is organized into four semi-overlapping layers: a goalie, defenders, forwards, and mid-fields. During play, each has a set of unique responsibilities, but they also have general overall responsibilities. Each of the layers function independently of others, that is, the forwards work together, as do the mid-fielders. The individual players in each layer can also function independently of each other. However, the team functions as a cohesive organization. All players are aware of their surroundings and the overall picture of the game and are capable of instantaneously reacting to dynamic changes on the playing field. In soccer, each player can see all 22 players and the ball. Each is capable of coordinating his actions. In combat the network is needed so that all players can "see the playing field" and coordinate their actions.

B. PROBLEM

If the Navy After Next is a force that is founded upon the principles of Network Centricity, then the U.S. Navy needs to pursue innovative methods to coordinate, operate and sustain joint forces in the future. These innovations must include the research and development of weapon systems that are themselves capable of operating in and contributing to a networked system. Decision makers, at all levels, will be challenged by the paradigm shift—operating as discrete systems functioning as one network. Similarly, analysts will require the capability to examine problems and provide useful and insightful analysis to the decision makers. They will need an analysis methodology that is responsive enough to keep up with the underlying warfare innovations.

1. Alternative to Complex Simulation for Exploratory Analysis

Although simulation is a very powerful and useful tool for solving some problems, it is not always the most appropriate method for others. Elaborate combat simulations, which rely upon high fidelity modeling, may not be responsive to rapidly changing technological advances in weapons systems and warfare concepts. The Navy After Next requires a simple model that is capable of capturing the innate properties of different weapon systems and allowing decision makers to understand the relationship and interaction between forces and pursue alternatives. If we can capture and model the essential characteristics of these weapon systems, we may be capable of providing easy, but limited, analysis.

The Hughes Salvo Model [Ref. 5] is such a model. It gives the analyst a simple method in which to evaluate the effects of salvo warfare between two opposing forces and allows simple insight into the broad characteristics of the battle. The model calculates the fraction of ships, or a force of identical ships, put out of action as a result of successful hits from an opposing ship or force.

2. Modeling Homogeneous and Heterogeneous Forces

The words homogeneous and heterogeneous come from the Greek words *homogenēs* and *heterogenēs*, which mean *same kind* and *other kind* [Ref. 6]. Homogeneous forces are those forces composed of the same kind of notional combat unit. Each unit displays the same combat characteristics, such as the rate of fire, rate of

defense, reconnaissance capability, survivability, etc. In contrast, a heterogeneous force is composed of "other kinds" of units, that is, combat units with different characteristics. A homogeneous force is a subset, or special case, of heterogeneous forces. A modeling technique to transform heterogeneous forces into homogeneous forces is to calculate the force average value for every parameter and assign those value to each unit. A second technique is to calculate the weighted sum of a parameter for each unit in the force and use that value for a force size of one. These techniques often yield acceptable approximations but have limited applicability.

The Hughes model uses a ship's offensive and defensive characteristics as parameters in order to express the effects of salvo warfare against an opposing ship. The model is scalable, allowing salvo warfare modeling between opposing forces of identical, i.e. homogeneous, ships. However, to capture the heterogeneous nature of the Navy After Next and to exploit the differences between various units in order to see how they interact with each other in a cooperative network, the assumptions of the Hughes Salvo Model can be too limiting. By developing a heterogeneous extension to the salvo model, this thesis offers a refinement that will enable the basic salvo model approach to be applied to future naval combat.

C. PURPOSE

The purpose of this thesis is to present a low-resolution model for the Navy After Next and demonstrates its potential use as a combat model for decision makers. Chapter II, reviews the Hughes Salvo Model and discusses its limitations with heterogeneous forces. It also introduces a heterogeneous extension of the Hughes Salvo Model and demonstrates its use. Chapter III examines the accuracy of the heterogeneous extension using a previous work, which used historical data to test the Hughes Salvo Model. Chapter IV examines a hypothetical campaign scenario and demonstrates how the heterogeneous salvo model can be used to aid in operational decision-making. Chapter V provides the readers with final thoughts and recommendations for future work.

THIS PAGE INTENTIONALLY LEFT BLANK

II. SALVO MODEL

This chapter reviews the Hughes Salvo Model and discusses its limitations with heterogeneous forces. It then introduces a heterogeneous extension of the Hughes Salvo Model and demonstrates its use. Presented are terms, used by both models, that are essential for the reader's understanding of the use of the specific model. This chapter argues the need for a flexible combat model, which is capable of exploiting the differences between the ships, or units, that compose a force. This chapter discusses the limitations of the homogeneous salvo model to fulfill this need and presents the heterogeneous salvo model as a suitable model for simple analysis of salvo warfare for the Navy After Next.

A. HUGHES SALVO MODEL

The Hughes salvo model is a simple mathematical methodology that captures the essential elements of salvo warfare for analysis. It is also a means "with which to compare the military worth of warship capabilities." [Ref. 5] The model provides a descriptive method of exploring the relationships between offensive, defensive, and staying power, and the number of units with those of an opposing force.

At the heart of the model is the salvo equation, which calculates the fraction of ships in a force that are placed out of action as a result of successful hits from an opposing force. In its simplest form, the salvo equation yields the fraction of hits all ships receive divided by the number of hits necessary to place a ship out of action. The result is the number of ships in the force put out of action by an enemy salvo of antiship missiles.

1. Definitions

a) Force

A group of naval ships that operate and fight together.

b) Unit

A unit is an individual ship in a force.

c) Salvo

A salvo is the number of shots fired as a unit of force in a discrete period of time.

d) Combat Potential

Combat Potential is a force's total stored offensive capability of an element or force measured in number of total shots available.

e) Combat Power

Also called Striking Power, is the maximum offensive capability of an element or force per salvo, measured in the number of hitting shots that would be achieved in the absence of degrading factors.

f) Scouting Effectiveness

Scouting Effectiveness is a dimensionless degradation factor applied to a force's combat power as a result of imperfect information. It is a number between zero and one that describes the difference between the shots delivered based on perfect knowledge of enemy composition and position and shots based on existing information [Ref. 7].

g) Training Effectiveness

Training effectiveness is a fraction that indicates the degradation in combat power due the lack of training, motivation, or readiness.

h) Distraction Factor

Also called chaff effectiveness or seduction, is a multiplier that describes the effectiveness of an offensive weapon in the presence of distraction or other soft kill. This multiplier is a fraction, where one indicates no susceptibility/complete effectiveness and zero indicates complete susceptibility/no effectiveness.

i) Offensive Effectiveness

Offensive effectiveness is a composite term made of the product of scouting effectiveness, training effectiveness, distraction, or any other factor which represents the probability of a single salvo hitting its target. Offensive effectiveness transforms a unit's combat potential parameter into combat power.

j) Defensive Potential

Defensive potential is a force's total defensive capability measured in units of enemy hits eliminated independent of weapon system or operator accuracy or any other multiplicative factor.

k) Defensive Power

Defensive power is the number of missiles in an enemy salvo that a defending element or force can eliminate.

l) Defender Alertness

Defender alertness is the extent to which a defender fails to take proper defensive actions against enemy fire. This may be the result of any inattentiveness due to improper emission control procedures, readiness, or other similar factors [Ref. 7]. This multiplier is a fraction, where one indicates complete alertness and zero indicates no alertness.

m) Defensive Effectiveness

Defensive effectiveness is a composite term made of the product of training effectiveness and defender alertness. This term also applies to any value that represents the overall degradation of a force's defensive power.

n) Staying Power

Staying power is the number of hits that a unit or force can absorb before being placed out of action.

2. Homogeneous Salvo Model

The two equations that make up the Homogeneous Salvo Model are:

$\Delta A = \dfrac{\beta' B - a_3' A}{a_1}$ and $\Delta B = \dfrac{\alpha' A - b_3' B}{b_1}$, where,

$$\beta' = (\sigma_B \tau_B \rho_A)\beta$$
$$\alpha' = (\sigma_A \tau_A \rho_B)\alpha$$
$$a_3' = (\delta_A \tau_A)a_3$$
$$b_3' = (\delta_B \tau_B)b_3$$

A = number of units in force A.
B = number of units in force B.
α = the number of well-aimed missiles fired by each A unit per salvo.
β = number of well-aimed missiles fired by each B unit per salvo.
a_3 = number of well-aimed attacking missiles eliminated by each A unit per salvo.
b_3 = number of well-aimed attacking missiles eliminated by each B unit per salvo.
a_1 = number of missiles required to place an A unit out of action.
b_1 = number of missiles required to place a B unit out of action.
σ_A = Scouting effectiveness of force A.
σ_B = Scouting effectiveness of force B.
δ_A = Defender alertness for force A.
δ_B = Defender alertness for force B.
τ_A = Training effectiveness for force A.
τ_B = Training effectiveness for force B.
ρ_A = Distraction factor for force A.
ρ_B = Distraction factor for force B.
ΔA = number of A units put out of action from B's salvo.
ΔB = number of B units put out of action from A's salvo.

The model assumes that offensive salvos are distributed uniformly across all of the defender's ships. Although a more descriptive distribution can be used, Hughes argues that in the past, when targets were in sight of each other, optimality of fire was never achieved and therefore the assumption of uniformity is sufficient for analysis [Ref. 5]. The model also assumes that a force's defense is perfect until it reaches its saturation point at which time it can no longer defend against additional salvos. A third assumption is that offensive combat power and defensive power are linear under damage, e.g., a ship with staying power of two hits that is hit by a single salvo has its offensive and defensive power reduce by one half.

B. HETEROGENEOUS SALVO MODEL

The CNAN project envisions a networked force, whose total combat and defensive power are widely distributed, as more advantageous than a force with combat and defensive power concentrated in only a few hulls. It envisions the Navy after Next as a mobile network of platforms, sensors, and weapons. Each node in this heterogeneous network represents one or more pieces of a weapon system, whether a platform, sensor, or weapon, and the network arcs are the links between nodes. Each node has its own characteristic parameters for offensive, defensive, and staying power. Each node may or

may not have some parameter that describes its scouting characteristics or some other types of combat multipliers. There may also be characteristic parameters that indicate the connectivity of a node and the quality of the arc. A node with no connection contributes nothing to the total force, while a node with only degraded arcs contributes a fraction of its capabilities. The optimality of arc configuration and the information provided is a topic for future analysis.

Historically, naval forces were typically divided into three categories of ships based upon the perceived mission. First was the battle fleet, whose prime responsibility was to destroy the enemy fleet. Second was the cruiser fleet, which raided enemy commerce and protected friendly commerce. Third was the flotilla of small craft, which fought in the littorals. [Ref. 8] There is not an easy one-for-one comparison with modern naval forces. A variety of ship types and classes have the capability to function across a number of mission areas. For example, large decked ships carry the individual fighting entities (aircraft, marines, special force, etc) that fight both enemy naval and ground forces. Aegis cruisers and destroyers have a dual role in the modern naval force by both engaging enemy fleets and protecting friendly warships. Fast patrol craft and special operations ships not only protect the littorals, but are also capable of engaging the enemy's battle force.

There are a few instances in modern naval combat where a portion of the battle force comprised of smaller units. During World War II, for example, the United States used PT boats in both theaters of war. The combat environment of the time necessitated an alternate approach to the traditional modes of naval warfare. In these smaller forces, the assets were capable of delivering powerful ordnance to high-valued enemy targets without subjecting capital ships to direct harm.

The complex and powerful warships of the current generation possess the most advanced technological devices and are capable of completing a variety of missions. Currently, United States naval ships have the ability to deliver an unmatched quantity of combat power to a wide variety of targets while simultaneously applying a large amount of defensive power against an enemy to counter his combat power. However, analysis demonstrates that homogeneous forces, those composed of identical units, display

instability when the staying power is small compared to the combat power [Ref. 7]. That is, if a force's combat potential is large relative to its survivability, then a few hits may result in the catastrophic loss of the entire force and underscores the need for a first, unanswered strike against an opposing force. To increase stability, there must be an increase in either the force's numbers or its unit staying power [Ref. 9].

Therefore, an alternate approach that increases the defensive or staying power is to distribute a fraction of the force's combat power in many individual units so that its loss costs only a small fraction of the force's combat potential. For example, if there are five ships, each with the capability to fire five missiles, the then force has a total combat power of 25. If one ship is damaged or unable to engage in battle, the force's combat power is reduced to 20. If, however, a fraction of missiles could be distributed to other units in the force, then the loss of one ship would only results in the loss of some fraction of the five missiles. The remaining missiles are then available for future engagements. Increasing the size of the force and distributing the combat and defensive potential in many smaller combat units achieve the same effect. In the extreme case 25 ships might be employed instead of five, each with the capability to fire one missile.

1. Assumptions and Definitions

By altering the Hughes Salvo Model [Ref. 5], it is possible to develop a low-resolution model of heterogeneous forces to support analysis of the CNAN force, as well as current forces, and their tactical use. However, there are two necessary assumptions to develop a simple variation of this model. First, as in the homogeneous model, the rate of loss for an A_i unit is linear in relation to the number of successful hits from all B_j enemy units attacking. Second, there is no direct synergism between systems, that is, attrition does not depend upon coordination between units of the same force. It seems reasonable that the first assumption should hold, but further analysis is necessary to investigate the validity of the second assumption. The model's core remains that the change in a force's size after a salvo exchange is equal to the total attrition caused by an opposing force.

In the homogeneous salvo equation, the characteristic values for the staying power, offensive potential and defensive potential for all entities are aggregated into three values that are used in the simple model. This generalization leads to a quick and broad analysis and it is fairly accurate when the various units are similar in capabilities, that is,

there is little difference between different units. However, as the differences in the ships' characteristics increase, the aggregated terms fail to capture the essence of any particular platform and the desirable tactics that each side should try to employ. To illustrate, say that a particular soccer goalkeeper has superior defensive capabilities, but no offense capability. The rest of the team has a low value for defense but a large value for offense. By aggregating the goalkeeper's values with those of the rest of the team and taking the average, the goalkeeper's defenses are artificially lowered while the rest of the team's defenses rise. A similar effect holds for the offensive capabilities. To better capture the contributions of each member of the team, or force, requires a method that separates the various characteristics while maintaining the simplicity of the salvo model.

In the homogeneous salvo equation, the number of hits sustained by a force is defined as the difference between the enemy's offensive power and the force's defensive power. For the purpose of exposition, let there be two forces, Blue and Red, each composed of truly homogeneous units. The battle between two ships is a trivial one-on-one battle. If, however, the battle is between a Blue force (of size n ships) and a single Red ship, then it is possible to view the battle as a series of n-sequential one-on-one battles. The number of hits sustained by the Red ship is then the sum of all the salvos from the n Blue ships minus the Red ship's defensive salvos. This is the numerator of the salvo equation. Now, a Blue ship can be hit if and only if the single Red ship targets and fires a salvo at it. Unlike the Red hits, the number of hits on the Blue force is the sum of hits from each one-on-one exchange with the Red ship. The third case is a battle between a force of n-Blue ships and a force of m-Red ships—an n-on-m battle. As in the second case, the total hits is the sum of n, one-on-m battles, where an individual Blue ship can be hit if and only if one or more of the m Red ships target and fire one or more salvos against it. The same is true for the total hits on the Red force. In all cases the reduction in force size, from the salvo equation, is the total number of hits sustained divided by the total staying power. If the two forces are not homogeneous, but are composed of different types of ships or weapon systems, then the change in the force size (ΔA) is sum of the results of all n, one-on-m engagements. By introducing two additional parameters, it is possible to modify the homogeneous salvo equation to allow analysis of two

heterogeneous forces, composed of different groups of similar weapon systems. The first parameter, *offensive targeting parameter*, is a number between zero and one that indicates the fraction of units in a particular group that engage an enemy group. In the case were each group is made up of single ship, then some fraction of the ship's combat potential is targeted against one of the opponent's ships. The second parameter, the *defensive targeting parameter*, is similar to the offensive targeting parameter. This number indicates the fraction of a group's defenses applied against the combat power of a specific enemy group.

2. Heterogeneous Salvo Model

The notion of an offensive and a defensive targeting parameter, used to divide a unit's or force's combat power among multiple enemy units or forces, is similar to the allocation term used when describing modeling heterogeneous forces using Lanchester type attrition models. [Ref. 10] Adding the two targeting parameters to the salvo equation and summing over all combinations gives the following generalized heterogeneous salvo equation for B attacking A.

$$
\begin{aligned}
\Delta A &= \sum_{i=1}^{m}\sum_{j=1}^{n}(\text{attrition of } A_i \text{ unit caused by } B_j) \\
&= \sum_{i=1}^{m}\sum_{j=1}^{n}\left(\frac{(\sigma_{ji}\tau_{ji}\rho_{ij})\beta_{ji}B_j - (\delta_{ij}\tau_{ij})\gamma_{ij}A_i}{\varsigma_i}\right) \\
&= \sum_{i=1}^{m}\sum_{j=1}^{n}\left(\frac{(\sigma_{ji}\tau_{ji}\rho_{ij})\beta_{ji}(\Psi_{ji}B_j) - (\delta_{ij}\tau_{ij})\gamma_{ij}(\Theta_{ij}A_i)}{\varsigma_i}\right)
\end{aligned}
$$

β_{ji} = Offensive combat potential of B_j units against A_j. {hits/shooting unit}
ψ_{ji} = Fraction of B_j units that engage A_i units. {[0,1]}
B_j = Number of B units of type j. {B_j units}
γ_{ij} = Defensive combat power of side A_i against B_j units. {shots /defending units}
Θ_{ij} = Fraction of A_i units that engage B_j units. {[0,1]}
A_i = Number of A units of type i. {A_i units}
ς_I = Staying power of A_i unit. {hits}
σ_{ji} = Scouting effectiveness of unit B_j against A_i. {[0,1]}
τ_{ji} = Training effectiveness of unit B_j against A_i. {[0,1]}
ρ_{ij} = Distraction factor of unit A_i against B_j. {[0,1]}
δ_{ij} = Defender alertness or readiness of unit A_i against B_j. {[0,1]}
ΔA = The number of A units put out of action .

As in the homogeneous model, the offensive and defensive effectiveness parameters, $(\sigma_{ji}\tau_{ji}\rho_{ij})$ and $(\delta_{ij}\tau_{ij})$, are composite indices [0,1] of scouting, training, and distraction factors that are used to degrade an attacker or defender's capabilities. For example, a unit consisting of four missiles, but is only 50% trained, can only apply two missiles against an opponent.

From this equation, the change in size of A_i unit is the sum of all simultaneous battles against the j enemy forces.

$$\Delta A_i = \sum_{j=1}^{n}\left(\frac{(\sigma_{ji}\tau_{ji}\rho_{ij})\beta_{ji}(\Psi_{ji}B_j - (\delta_{ij}\tau_{ij})\gamma_{ij}(\Theta_{ij}A_i))}{\varsigma_i}\right), \forall i$$

3. Heterogeneous Salvo Model in Matrix Notation

Letting O_{ji} equal the offensive coefficients for B_j attacking A_i and letting D_{ij} equal the defensive coefficients for A_i units defending against B_j, then, expanding and rearranging terms, the Salvo Equation can be written as

$$\Delta A_i = (O_{1i}B_1 + O_{2i}B_2 + \ldots + O_{ji}B_j) - (D_{i1} + D_{i2} + \ldots + D_{ij})A_i.$$

To illustrate this transformation, let i=1 and j=2.

$$\begin{aligned}\Delta A_1 &= \frac{(\sigma_{11}\tau_{11}\rho_{11})\beta_{11}(\Psi_{11}B_1) + (\sigma_{21}\tau_{21}\rho_{21})\beta_{21}(\Psi_{21}B_2)}{\varsigma_1} - \frac{(\delta_{12}\tau_{12})\gamma_{12}(\Theta_{12}A_1) + (\delta_{11}\tau_{11})\gamma_{11}(\Theta_{11}A_1)}{\varsigma_1} \\ &= \frac{(\sigma_{11}\tau_{11}\rho_{11})\beta_{11}(\Psi_{11}B_1) + (\sigma_{21}\tau_{21}\rho_{21})\beta_{21}(\Psi_{21}B_2)}{\varsigma_1} - \frac{(\delta_{11}\tau_{11}\gamma_{11}\Theta_{11} + \delta_{12}\tau_{12}\gamma_{12}\Theta_{12})(A_1)}{\varsigma_1} \\ &= O_{11}B_1 + O_{21}B_2 - (D_{11} + D_{12})A_1\end{aligned}$$

where $O_{11} = (\sigma_{11}\tau_{11}\rho_{11}\beta_{11}\Psi_{11})$, $O_{21} = (\sigma_{21}\tau_{21}\rho_{21}\beta_{21}\Psi_{21})$, $D_{11} = (\delta_{11}\tau_{11}\gamma_{11}\Theta_{11})$, $D_{12} = (\delta_{12}\tau_{12}\gamma_{12}\Theta_{12})$

Generalizing, the Salvo Equation takes the matrix form

$$\mathbf{OB^T - DA = \Delta A}$$

O = Matrix of offensive coefficients =

$$\begin{bmatrix} O_{11} & \cdots & O_{j1} \\ \vdots & \ddots & \vdots \\ O_{1i} & \cdots & O_{ji} \end{bmatrix}, \text{where } O_{ji} = \sigma_{ji} \otimes \tau_{b_{ji}} \otimes \rho_{ji}^{T} \otimes \beta_{ji} \otimes \Psi_{ji} \otimes \left[1/\varsigma_{ji}\right]^{T}$$

D = Matrix of defensive coefficients =

$$\begin{bmatrix} D_{11} + \ldots + D_{1j} & 0 & 0 & 0 \\ 0 & D_{21} + \ldots + D_{2j} & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & D_{i1} + \ldots + D_{ij} \end{bmatrix}, \text{where } D_{ij} = \delta_{ij} \otimes \tau_{a_{ij}} \otimes \gamma_{ij} \otimes \Theta_{ij} \otimes \left[1/\varsigma_{ij}\right]^{T}$$

B = Matrix of B-units.

A = Matrix of A-units.

C. CHAPTER SUMMARY

The heterogeneous model is an extension of the homogeneous salvo model that gives the analysts a greater degree of flexibility to examine and model the unique characteristics of heterogeneous forces. With this increase in flexibility is an increase in the number of parameters that must be accounted for and processed. In the homogeneous salvo model, the modeler had two forces, each with five to seven parameters. In the heterogeneous model, the worst case could result in $m \times n$ different mini-engagements, each with five to seven parameters. A solution to this complex accounting is the transformation of the heterogeneous salvo model into matrix notation. While providing a compact form to use, it also exploits the existing body of theorems and proofs associated with matrix mathematics.

III. HETEROGENEOUS MODEL EXAMINATION WITH HISTORICAL DATA

This chapter examines the accuracy of the heterogeneous salvo model by comparing its results with those from a previous work that tested Hughes' salvo model. That previous work done by Thomas Beall, [Ref. 11] gathered data on various naval battles and, using that information as model inputs, calculated the resulting combat power loss of each naval force. This chapter examines the accuracy of the heterogeneous salvo model's calculations by comparing them against those of Beall's thesis.

A. HISTORICAL DATA VALIDATION OF NAVAL BATTLE MODEL

Beall's work [Ref. 11] used data from historical naval battles to verify his own computer model, which was based upon the Hughes Salvo Model. Through data analysis, Beall captured the mathematical essences of 14 World War I and II naval battles. He then used that data to simulate the battles and to compare their results with actual outcomes.

1. Background on Beall Analysis

Beall used historical data in survivability analysis to develop a method to calculate the staying power value for any surface warships. His goal was to model the number of hits required to inflict a firepower kill, i.e., sufficient damage to prevent a unit from contributing further combat power as a function of its displacement. The standard weapon used through out his analysis was the 1000-pound heavy case bomb, which had an equivalent explosive weight of 660 pounds of TNT. The resulting equation, $N = 0.070369 \times displacement_i^{1/3}$, is used to provide an appropriate value that describes the staying power of a warship.

To determine the offensive potential, Beall first separated weapon and platform characteristics into two combat potential categories: continuous combat power and pulsed combat power. Using the "ballistic mortar strength" and rate of fire, Beall was able to calculate the explosive weight per minute that a particular gun system on a specific platform was capable of firing. He calculated the combat potential of a specific continuous weapon as the amount of Thousand Pound Bomb Equivalent (TPBE) that can

be fired by a gun per minute, calculated by $\frac{\text{weight}}{660\text{ lbs}}\times 2.5$. The constant, 2.5, gives added weight to a gun's shell over that of a bomb of equal explosive weight due to the additional kinetic energy that the shell imparts upon impact [Ref. 11]. Beall derived this constant through survivability analysis. The aggregated continuous combat potential for a platform is the sum of all gun systems on the platform. The aggregated continuous combat potential for an entire group of ship is the sum of all the individual ship's combat continuous combat potential.

Battles involving pulsed weapons (torpedo salvos or air wing strikes) required a slightly different approach to calculate the combat power. A group's total pulsed combat power is calculated using the following equation:

$$\frac{\text{TNT equiv. weight}}{660\text{ lbs}}\times \text{Number of weapons per salvo}\times \text{Wtp}.$$

The parameter, Wtp, is a scalar used to give torpedoes additional combat power since their destruction occurs at or below the waterline and inflicts more damage than bombs. In Beall's analysis and this thesis, bombs have a Wpt value of 1.0 and torpedoes have a value of 1.25. Beall calculated the total group pulsed combat potential and weapon effectiveness by weapon type for each group in a force. His model, however, does not distinguish group pulsed power. Rather, it uses a single value for the total pulsed power fired by all groups. This eliminates the requirement to track specific pulsed weapons and their effect.

In order to compare the analytical results with the historical data, Beall expressed the outcome as a percent of firepower lost by each force. Since it was not possible to attribute specific damage done to a ship with its firepower loss, a lower and upper bound was calculated for the entire force. The results of Beall's analysis are then compared with the combat power loss interval. The lower limits of this loss interval "represent the percent of a force's [combat power] carried by all of the force's platforms suffering at least a firepower kill during a given battle. The upper limits represent the percent of [combat power] carried by all of the force's platforms suffering at least some damage in the battle." [Ref.11] In this chapter, the calculated firepower losses from Beall's analysis

are compared with the calculations from the heterogeneous salvo model, for three of the 14 battles.

2. Setup of the Heterogeneous Salvo Model for Analysis

This chapter uses *Microsoft® Excel* [Ref. 12] spreadsheet as the calculating device to implement the heterogeneous salvo model. It was chosen because it is widely available and requires only minimal programming skills to setup the model. This product does have some limitation on its usefulness and so the readers are urged to implement the heterogeneous salvo model by whatever method aids analysis.

To implement the heterogeneous salvo model so that it would replicate the results of Beall's data for gunfire, it was necessary to make two modifications. As in the Beall's simulation, the implementation of the heterogeneous salvo model can be structured to calculate the results of "salvos" in one-minute increments.

The first modification transforms Beall's homogeneous, continuous fire data for direct implementation in the heterogeneous salvo model. Beall divides the opposing forces into separate groups of ships that operate together throughout a battle. To determine the combat power for each group, he summed the combat power of every ship in that group. This total then becomes the parameter value used in the model. This process was also done to calculate each group's staying power.

Using Beall's aggregated data, the groups are disaggregated, or separated into individual ship with equal weights. This transforms any single battle event from a one-on-one battle to an m-on-n battle, where all m or n ships have the same characteristics. To illustrate, say Group A is composed of five ships with a combat power of 6.0, 1.0, 1.0, 1.0, and 1.0 for a total combat power of 10.0 for the group. The heterogeneous model transforms Group A back into five units each with a combat power of 2.0. This is an artifact of the process of checking the model's accuracy with Beall's analysis. If this was an independent application of the heterogeneous model, it would use two groups. Group A would have one ship with a combat power of 6.0 and Group B would have four ships with a combat power of 1.0 each. Additionally, pulsed power weapons are not associated with the firing platform from which they were fired. Instead, each pulsed weapon type serves as an independent B_j unit where the combat power parameter, β_{ji}, remains constant

throughout the battle. The size of pulsed power weapons varies with each salvo exchange and is calculated independently using the historical data.

The second modification to the heterogeneous salvo model is to determine the offensive effectiveness for the two forces in any given battle. Through research, Beall was able to determine the total number of hits scored by groups of ships during a battle. This number was divided by either the total number of rounds fired, if known, or the expected total number of rounds fired, which is calculated based upon the composition of weapons and their characteristic rate of fire. Throughout this chapter the offensive effectiveness is substituted for $(\sigma_{ji}\tau_{ji}\rho_{ji})$ in the heterogeneous salvo model and labeled ε_{ji}.

Beall's data is divided into three categories that are based upon the type of weapon fire present in the battle. First, "continuous fire battles" are those in which gunfire was applied continuously by each side as the primary means of inflicting damage. Second, "pulsed fire battles" are those in which effective combat power was applied in pulses, either by using aerial bombs or torpedoes. Third, "mixed fire battles" are those in which both continuous and pulsed fire was used in the battle. [Ref. 11] Of the 14 battles examined, the analysis of three representative battles are presented: The Battle of Coronel (01 November 1914), The Battle of Coral Sea (07 May 1942), and The Battle of Savo Island (08 August 1942). In the following sections, the historical summary of the battle and the calculations of parameters for each force are presented. The results of the heterogeneous salvo model are presented and compared against the data gathered by Beall.

B. BATTLE OF CORONEL

The Battle of Coronel is a World War I naval engagement between three British ships (*Good Hope*, *Monmouth*, and *Glasgow*) and four German ships (*Scharnhorst*, *Gneisenau*, *Leipzig*, and *Dresden*). The *Scharnhorst* and *Gneisenau* are the first ships to open fire at *Good Hope* and *Monmouth*; the three British ships soon afterwards return fire. *Dresden* and *Leipzig* open fire on *Glasgow*, driving her out of the engagement. At the end of the battle, both *Good Hope* and *Monmouth* are sunk, while *Glasgow*, *Scharnhorst*, and *Gneisenau* were damaged. [Ref. 11]

1. Beall Analysis of The Battle of Coronel

In The Battle of Coronel, Beall estimated the weapons effectiveness for *Scharnhorst* and *Gneisenau* by calculating the number of hits the two ships scored (50) in the 28 minutes of fire and dividing it by the total number of shells fired (1800). Similarly the *Dresden* and *Leipzig* each scored five hits and fired a total of 400 shells in two minutes. *Glasgow* fired for 15 minutes and is estimated to have fired 210 shells with six hits. Table 1 is a summary of the data gathered by Beall for The Battle of Coronel.

Ships (by group)	Total Group Combat Power	Total Group Combat Effectiveness	Total Staying Power	Duration of Fire (Minutes)
(1) *Good Hope* *Monmouth*	7.27	0.00	3.21	0
(2) *Glasgow*	0.42	0.028	1.23	15
(3) *Scharnhorst* *Gneisenau*	4.32	0.028	3.330	28
(4) *Leipzig* *Dresden*	4.33	0.012	2.23	2

Table 1. Summary of Beall Data for the Battle of Coronel

Beall calculated a combat power loss interval of 94.18% - 100.00% for the British force and a loss interval of 0.00% - 49.92% for the German force. His simulation calculated a 94.77% loss of combat power for the British force and 2.67% for the German force. Since these firepower loss values lie within the corresponding loss intervals, Beall concluded that the simulation results are valid.

2. Heterogeneous Salvo Model Analysis of The Battle of Coronel

Using the heterogeneous salvo model, the number of ships remaining after each of the two events is calculated. In the first event there are a total of six simultaneous one-on-one engagements: *Scharnhorst* vs. *Good Hope*, *Scharnhorst* vs. *Monmouth*, *Gneisenau* vs. *Good Hope*, *Gneisenau* vs. *Monmouth*, *Glasgow* vs. *Scharnhorst*, and *Glasgow* vs. *Gneisenau*. Since parameters were derived from the average of the corresponding parameter in the Beall data, we expect the results from *Scharnhorst* vs.

Good Hope to be the same as *Gneisenau* vs. *Good Hope*, and similarly for the other group pairings. In the second event there are four simultaneous one-on-one engagements: *Dresden* vs. *Glasgow*, *Leipzig* vs. *Glasgow*, *Glasgow* vs. *Dresden*, and *Glasgow* vs. *Leipzig*.

From the historical data provided by Beall, the first event to takes place in The Battle of Coronel is an engagement where *Scharnhorst* and *Gneisenau* open fire on *Good Hope* and *Monmouth* for 18 minutes. The inputs for the heterogeneous salvo model are:

$$
\begin{aligned}
&\beta_{11} = \beta_{21} = \beta_{31} = 2.16\\
&\beta_{12} = \beta_{22} = \beta_{32} = 2.16\\
&\beta_{13} = \beta_{23} = \beta_{33} = 2.165\\
&\beta_{14} = \beta_{24} = \beta_{34} = 2.165\\
&\varsigma_1 = \varsigma_2 = 1.605\\
&\varsigma_3 = \varsigma_4 = 1.23\\
&\varepsilon_{11} = \varepsilon_{12} = \varepsilon_{21} = \varepsilon_{22} = \varepsilon_{31} = \varepsilon_{32} = 0.028\\
&\varepsilon_{31} = \varepsilon_{32} = \varepsilon_{41} = \varepsilon_{42} = \varepsilon_{33} = \varepsilon_{43} = 0.012
\end{aligned}
$$

For purpose of exposition the calculations involved in one time step are demonstrated. The parameter ψ_{ji}, the fraction of B_j units that engage A_i, varies with time depending on whether that B_j units actually fired a weapon during that time interval. The following calculation is for A_1, *Good Hope*, for first time step:

$$
\Delta A_1 = \sum_{j=1}^{n}\left(\frac{\varepsilon_{ji}\beta_{ji}\left(\Psi_{ji}B_j\right)}{\varsigma_i}\right) = \frac{\varepsilon_{11}\beta_{11}\Psi_{11}B_1}{\varsigma_1} + \frac{\varepsilon_{21}\beta_{21}\Psi_{21}B_2}{\varsigma_1} + \frac{\varepsilon_{31}\beta_{31}\Psi_{31}B_3}{\varsigma_1} + \frac{\varepsilon_{41}\beta_{41}\Psi_{41}B_4}{\varsigma_1} =
$$

$$
\Delta A_1 = \frac{0.028\times 2.16\times 0.5\times 1.0}{1.605} + \frac{0.028\times 2.16\times 0.5\times 1.0}{1.605} +
$$

$$
\frac{0.012\times 2.165\times 0\times 1.0}{1.605} + \frac{0.012\times 2.165\times 0\times 1.0}{1.605} =
$$

$$
\Delta A_1 = 0.037
$$

Since *Scharnhorst* and *Gneisenau* engage both *Good Hope* and *Monmouth* simultaneously during the first minute, their corresponding ψ_{ji} value is 0.5. An equivalent calculation is made for A_2, *Monmouth*, with identical results. No other ships fired during the first minute so all other calculations are expected to resemble the following:

$$\Delta A_3 = \sum_{j=1}^{n}\left(\frac{\varepsilon_{ji}\beta_{ji}\left(\Psi_{ji}B_j\right)}{\varsigma_i}\right) = \frac{\varepsilon_{13}\beta_{13}\Psi_{13}B_1}{\varsigma_3} + \frac{\varepsilon_{23}\beta_{23}\Psi_{23}B_2}{\varsigma_3} + \frac{\varepsilon_{33}\beta_{33}\Psi_{33}B_3}{\varsigma_3} + \frac{\varepsilon_{43}\beta_{43}\Psi_{43}B_4}{\varsigma_3} =$$

$$\Delta A_3 = \frac{0.028\times 2.16\times 0\times 1.0}{0.42} + \frac{0.028\times 2.16\times 0\times 1.0}{0.42} + \frac{0.012\times 2.165\times 0\times 1.0}{0.42} + \frac{0.012\times 2.165\times 0\times 1.0}{0.42} =$$

$$\Delta A_3 = 0.00$$

3. Conclusions

After 28 calculations, representing the 28 minutes of combat, the heterogeneous salvo model calculates that the *Good Hope* and *Monmouth* are both destroyed, i.e., $\Delta A_i \cong 1$ for both ships. The model calculates $\Delta A_i = 0.084$ for the Glasgow, resulting in a total remaining force size of 0.916 ships for the British force. The model calculates a $\Delta B_j = 0.053$ for the *Scharnhorst* and *Gneisenau*, reducing the German force to 3.894 ships. The corresponding firepower loss is 94.76% for the British force and 2.67% for the German force. When comparing the heterogeneous salvo model calculations with those from Beall's simulation, it is shown that the results are identical, which is what was expected.

C. BATTLE OF CORAL SEA

The Battle of Coral Sea examines the effects of pulsed weapons using the heterogeneous salvo model. Beall considered only engagements between aircraft carriers, using airplanes as the delivery platform for combat power. The U.S. force consisted of the two aircraft carriers, *Lexington* and *Yorktown*. The *Lexington* displaced 43055 tons and carried 17 Dauntless, 17 Dauntless Scouts, and 11 Devastators. The Dauntless was capable of carrying one 1000lb HC bomb, the Scout carried one 500lb HC bomb, and the Devastator carried one 22.4" torpedoes. The *Yorktown* displaced 25484 tons, carrying 17 Dauntlesses, 17 Dauntless Scouts, and 10 Devastators. The Japanese force consisted of the aircraft carriers *Shokaku* and *Zuikaku*. The *Shokaku* displaced 32105 tons, carrying 17 Vals and 13 Kate airplanes. The Vals were armed with one 250kg SAP bomb and the Kate carried one 18" torpedoes. The *Zuikaku* displaced 32105 tons and carried 16 Vals and 12 Kates. [Ref. 11]

The Battle of Coral Sea began with the U.S. carriers launching an air strike of 28 Dauntless and 20 Devastators, and the Japanese launching 33 Vals and 18 Kates at the U.S. carriers. In the first event, 24 Dauntless and 9 Devastators survive to attack the Japanese force, scoring 2 hits on the *Shokaku* with 1000lb bombs. In the second event, the Japanese strike force hits the *Lexington* with two 250kg SAP bombs and two 18" torpedoes, and hits the *Yorktown* with one 250kg. SAP bomb. In the final event, a second U.S. strike force attacked the *Shokaku*, scoring one hit with one 1000lb HC bomb. At the conclusion of battle, the *Yorktown* was sunk and the *Shokaku* suffered a firepower kill. [Ref. 11]

1. Beall Analysis of the Battle of Coral Sea

With the introduction of the airplane as delivery platforms for bombs and torpedoes, this battle resembles what Hughes refers to as salvo warfare [Ref. 5]. In this battle, the aircraft carriers have no offensive power other than the aircraft and weapons they carry, but are themselves the targets of enemy fire. Beall refers to this combination of weapon and delivery platform as pulsed power. The pulsed power effectiveness is calculated as a function of both the aircraft's survival and the bomb's detonation. In this battle 46 Dauntlesses were launched and three hits were attributed to this airplane type. This gave the 1000lb bomb an effectiveness parameter of 0.065 per hit. Similarly, the Japanese launched 33 Vals, scoring three hits, had an effectiveness of 0.091. Additionally, 18 Kates launched with two hits, giving the Japanese torpedo an effectiveness of 0.111. [Ref. 11] Table 2 is a summary of the data gathered by Beall for the Battle of Coral Sea.

Since the *Lexington* was lost and the *Yorktown* was damaged, Beall calculated the loss interval for the U.S. forces as 50.48% - 100.00%. The Japanese carrier *Shokaku* was damaged during the battle, resulting in a loss interval of 51.90% - 51.90%. Beall's simulation calculated the U.S. force's firepower loss as 53.35% and 51.90% for the Japanese force. Since both results fell within the combat power loss intervals, the model was assumed correct.

Ships (by group)	Total Group Combat Power	Total Group Combat Effectiveness	Total Ship Staying Power
Lexington *1000lbs* *500lbs* *torpedo*	17.00 7.91 8.34	0.065 0.000 0.000	2.42
Yorktown *1000lbs* *500lbs* *torpedo*	17.00 7.91 7.58	0.065 0.000 0.000	2.07
Shokaku *250kg* *torpedo*	3.68 12.10	0.091 0.111	2.42
Zuikaku *250kg* *torpedo*	3.46 11.17	0.091 0.111	2.24

Table 2. Summary of Beall Data for the Battle of Coral Sea.

2. Heterogeneous Salvo Model Analysis of the Battle of Coral Sea

The Battle of Coral Sea demonstrates a venue for which heterogeneous salvo model was intended. In this battle there are three separate salvo exchanges between the U.S. and Japanese forces. The first exchange is between the U.S. aircraft against the Japanese aircraft carrier *Shokaku.* The second exchange takes place when Japanese planes from both aircraft carriers attack the two U.S. carriers. The last exchange is between the aircraft from the *Lexington* and the *Shokaku.*

In the first exchange, 24 Dauntlesses and 9 Devastators survived to attack the Japanese ships. The combat power of the Dauntlesses is as 1.00 TPBE/pulse. Beall does not use the combat power of the 9 Devastators (0.785 each) because they had no effect in the model. For completeness, the Devastators are included in the heterogeneous salvo model, with a combat effectiveness of 0.0 TBPE/minute. This prevents the airplanes and their payloads from contributing any combat power against any target. Table 3 gives a summary of the pertinent parameter used in the heterogeneous salvo model.

Unit	B_j	ε_{ji}	β_{ji}	ζ_i
(1) *Lexington*	1	0.00	0.00	2.42
(2) *Yorktown*	1	0.00	0.00	2.07
(P1) *Dauntless*	m	0.065	1.00	0.00
(P2) *Dauntless Scouts*	m	0.000	0.466	0.00
(P3) *Devastators*	m	0.00	0.758	0.00
(1) *Shokaku*	1	0.00	0.00	2.42
(2) *Zuikaku*	1	0.00	0.00	2.24
(P1) *Vals*	n	0.091	0.216	0.00
(P2) *Kates*	n	0.111	0.931	0.00

Table 3. Summary of Coral Sea Data for the Heterogeneous Salvo Model

The first event in the battle is a 2 x 2 engagement between the two U.S. carriers and two Japanese carriers, but reduces to one-on-one battle between the Dauntlesses and the *Shokaku*. To demonstrate the results of the first event of this battle, the calculations for Japanese attrition and the resulting reduction of force size are presented.

$$\Delta B_1 = \sum_{j=1}^{5}\left(\frac{\varepsilon_{ji}\beta_{ji}\left(\Psi_{ji}A_j\right)}{\varsigma_i}\right) =$$

$$\frac{\varepsilon_{11}\beta_{11}\Psi_{11}A_1}{\varsigma_1} + \frac{\varepsilon_{21}\beta_{21}\Psi_{21}A_2}{\varsigma_1} + \frac{\varepsilon_{31}\beta_{31}\Psi_{31}A_3}{\varsigma_1} + \frac{\varepsilon_{41}\beta_{41}\Psi_{41}A_4}{\varsigma_1} + \frac{\varepsilon_{51}\beta_{51}\Psi_{51}A_5}{\varsigma_1}$$

$$\Delta B_1 = \frac{0.000\times 0.000\times 0\times 0}{2.42} + \frac{0.000\times 0.000\times 0\times 0}{2.42} + \frac{0.065\times 1.0\times 1.0\times 24.0}{2.42} +$$

$$\frac{0.000\times 0.758\times 0.0\times 9.0}{2.42} + \frac{0.000\times 0.000\times 0\times 0}{2.42}$$

$$\Delta B_1 = 0.645$$

$$B_1 = 0.355$$

$$\Delta B_2 = \sum_{j=1}^{5}\left(\frac{\varepsilon_{ji}\beta_{ji}\left(\Psi_{ji}A_j\right)}{\varsigma_i}\right) =$$

$$\frac{\varepsilon_{12}\beta_{12}\Psi_{12}A_1}{\varsigma_2}+\frac{\varepsilon_{22}\beta_{22}\Psi_{22}A_2}{\varsigma_2}+\frac{\varepsilon_{32}\beta_{32}\Psi_{32}A_3}{\varsigma_2}+\frac{\varepsilon_{42}\beta_{42}\Psi_{42}A_4}{\varsigma_2}+\frac{\varepsilon_{52}\beta_{52}\Psi_{52}A_5}{\varsigma_2}$$

$$\Delta B_2 = \frac{0.000\times0.000\times0\times0}{2.42}+\frac{0.000\times0.000\times0\times0}{2.42}+\frac{0.065\times1.0\times0.0\times24.0}{2.42}+$$
$$\frac{0.000\times0.758\times0.0\times9.0}{2.42}+\frac{0.000\times0.000\times0\times0}{2.42}$$

$$\Delta B_2 = 0.000$$
$$B_2 = 1.00$$

$$\Delta B = 0.645$$
$$B = 1.355$$

The heterogeneous model calculates the *Shokaku* and its combat power, if it had any, are reduced by 64.5%. Since no U.S. aircraft attack the *Zuikaku*, the model, as expected, calculates no reduction in its size. The second exchange involves the 33 Japanese Vals attacking both U.S. carriers, while 25 Kates torpedo the *Lexington*. The model calculates 90.3% attrition to the *Lexington*, i.e. $\Delta A_1 = 0.903$, and 15.7% attrition to the *Yorktown*, i.e., $\Delta A_2 = 0.157$. The heterogeneous salvo equation calculation for the *Yorktown's* attrition is provided below. The equation for ΔA_2 is similar except $\psi_{42} = 0.0$ and $\varsigma_2 = 2.07$.

$$\Delta A_1 = \sum_{j=1}^{4}\left(\frac{\varepsilon_{ji}\beta_{ji}\left(\Psi_{ji}B_j\right)}{\varsigma_i}\right) = \frac{\varepsilon_{11}\beta_{11}\Psi_{11}B_1}{\varsigma_1}+\frac{\varepsilon_{21}\beta_{21}\Psi_{21}B_2}{\varsigma_1}+\frac{\varepsilon_{31}\beta_{31}\Psi_{31}B_3}{\varsigma_1}+\frac{\varepsilon_{41}\beta_{41}\Psi_{41}B_4}{\varsigma_1}$$

$$\Delta A_1 = \frac{0.000\times0.000\times0\times0}{2.42}+\frac{0.000\times0.000\times0\times0}{2.42}+\frac{0.091\times0.216\times0.5\times33.0}{2.42}+$$
$$\frac{0.111\times0.931\times1.0\times18}{2.42}$$

$$\Delta A_1 = 0.903$$
$$A_1 = 0.097$$

In the last exchange, four U.S. Dauntlesses and 11 Devastators attack the *Shokaku*. Without demonstrating further calculations or details, the heterogeneous salvo equation calculates $\Delta B_1 = 0.107$, which reduces the fraction size of the *Shokaku* to 0.248.

3. Conclusions

Comparing the heterogeneous salvo model's calculation for the fractional combat power loss, it is shown that the calculated results lie within Beall's loss interval. The model calculates that $A_1 = 0.097$ and $A_2 = 0.843$ at the end of the battle. This corresponds to a combat power loss of 53.84%. The model calculates that $B_1 = 0.248$ and $B_2 = 1.0$ at the end of the battle, which corresponds to a combat power loss of 39.00%. The difference in the combat power loss for the Japanese force results from a difference in interpretation of the *Shokaku* results. The heterogeneous salvo model calculates that there is a 75.2% reduction in *Shokaku's* size, or combat power. Beall claims that the *Shokaku* suffered a firepower kill, which reduces its combat power to 0.0. If a 75.2% firepower reduction is sufficient criteria to claim a firepower kill, then the heterogeneous salvo model results fall within the computed intervals.

D. BATTLE OF SAVO ISLAND

The Battle of Salvo Island is a mixed battle, combining both continuous and pulsed power weapons, between the U.S. and Japanese naval forces. The U.S. force was composed of 15 ships, divided into seven groups. The Japanese force had eight ships in to two groups. In addition to their main battery guns, both forces had pulsed power systems in the form of torpedoes. The U.S. force used the MK15 21" torpedo and the British MKIX 21" torpedo. The Japanese force used both the 24" TYPE 93 and 21" 6 YR TYPE torpedoes. [Ref. 11]

The battle consists of a sequence of continuous fire engagements, with pulsed power engagements interspersed throughout. The battle began with a simultaneous pulsed and continuous fire attack by the Japanese force against the *Chicago* and *Canberra*. Two torpedoes and 24 shells struck the *Canberra*; the *Chicago* was hit by one torpedo. The Japanese force then opened fire with guns and shot 16 torpedoes at the *Vincennes*, *Astoria*, and *Quincy*, scoring numerous hits by shells and torpedoes. The *Vincennes*, *Astoria*, and *Quincy* return fire at *Aoba*, *Kako*, *Kinugasa*, *Furutaka*, and *Chokai*, scoring several hits. Finally, the U.S. ships *Blue* and *Ralph Talbot* engaged the Japanese force, which returned fire, hitting *Ralph Talbot* with four shells.

1. Beall Analysis of the Battle of Savo Island

In the Battle of Savo Island, Beall divided the U.S. force into seven groups and the Japanese force into two groups. The first U.S. group was composed of *Vincennes*, *Astoria*, and *Quincy* and fired 107 shells in one minute, scoring a total of four hits. The only other ships to fire effectively were *Blue* and *Ralph Talbot*, firing 385 shells in two minutes and scoring one hit. The U.S. force fired no torpedoes with effectiveness. The five Japanese cruisers fired a total of 1020 shells in five minutes scoring 92 hits. The entire Japanese force fired 61 torpedoes scoring seven hits. No other Japanese ship fired with effect in this battle. [Ref. 11] Table 4 is a summary of the parameters calculated and used by Beall in his simulation.

Ships (by group)	Total Group Combat Power	Total Group Combat Effectiveness	Total Ship Staying Power
(1) *Vincennes, Astoria, Quincy*	9.03	0.037	4.89
(2) *Helm, Wilson*	4.17	0.00	1.84
(3) *Chicago, Canberra*	6.38	0.00	3.31
(4) *Patterson, Bagley*	4.17	0.00	1.84
(5) *San Juan, Hobart*	11.70	0.00	2.90
(6) *Monssen, Buchanon*	5.21	0.00	1.88
(7) *Blue, Ralph Talbot*	4.17	0.003	1.84
(P1) *MK15 torpedo*	112.19	0.00	N/A
(P2) *MKIX torpedo*	5.51	0.00	N/A
(1) *Aoba, Kako, Kinugasa, Furutaka, Chokai*	11.85	0.090	7.88
(2) *Tenryu, Tubari, Yunagi*	2.28	0.00	3.02
(P1) *Type 93 torpedo*	56.02	0.115	N/A
(P2) *6 year type torpedo*	8.33	0.00	N/A

Table 4. Summary of Beall Data for the Battle of Savo Island

In the mixed battle, Beall calculated a combat power loss interval for both the continuous fire weapons and the pulsed power weapons. The *Vincennes*, *Astoria*, *Quincy*, *Canberra* and *Ralph Talbot* were lost, the *Chicago* suffered a firepower kill, and the *Patterson* was heavily damaged. Beall calculated a combat power loss interval of 38.89%-43.60% for continuous fire weapons and 10.49%-21.18% for pulsed weapons. The Japanese cruisers *Kinugasa* and *Chokai* were damaged, resulting in a loss interval of 0.00%-39.47% for continuous fire weapons and 0.00%-37.31% for pulsed weapons. [Ref. 11] Beall's simulation calculated a 39.77% continuous combat power loss and

12.28% pulsed power loss for the U.S. force, both of which lie within the computed interval. The Japanese force had a 0.16% continuous combat power loss and 0.16% pulsed power loss, which was also within the computed interval.

2. Heterogeneous Salvo Model Analysis of the Battle of Savo Island

The Battle of Savo Island examines the heterogeneous salvo model's capability to calculate hits and force size reductions for different types of weapons simultaneously. Since continuous combat power has effect over time, the model's implementation is similar to that of the Battle of Coronel. Force reduction, again, is calculated in one-minute intervals of time. At each interval, the model calculates the effects of all continuous fire and any pulsed weapons that impact during that interval. Table 5 lists the heterogeneous salvo model parameters for the Battle of Savo Island. The parameters, Ψ_{ji}, for the fraction of A_i units that engage B_j are not listed in this table since their value varies with time.

Unit	B_j	ε_{ji}	β_{ji}	ζ_i
Vincennes	1	0.037	3.01	1.63
Astoria	1	0.037	3.01	1.63
Quincy	1	0.00	3.01	1.63
Helm	1	0.00	2.085	0.92
Wilson	1	0.00	2.085	0.92
Chicago	1	0.00	3.190	1.655
Canberra	1	0.00	3.190	1.655
Patterson	1	0.00	2.085	0.92
Bagley	1	0.00	2.085	0.92
San Juan	1	0.00	5.85	1.45
Hobart	1	0.00	5.85	1.45
Monssen	1	0.00	2.605	0.94
Buchanon	1	0.00	2.605	0.94
Blue	1	0.003	2.085	0.92
Ralph Talbot	1	0.003	2.085	0.92
MK15 torpedo	n	0.091	1.558	N/A
MKIX torpedo	n	0.111	1.377	N/A
Aoba	1	0.09	2.37	1.576
Kako	1	0.09	2.37	1.576
Kinugasa	1	0.09	2.37	1.576
Furutaka	1	0.09	2.37	1.576
Chokai	1	0.09	2.37	1.576
Tenryu	1	0.00	0.76	1.007
Tubari	1	0.00	0.76	1.007
Yunagi	1	0.00	0.76	1.007
Type 93 torpedo	n	0.115	2.001	N/A
6 year type torpedo	n	0.00	0.833	N/A

Table 5. Summary of Savo Island Data for the Heterogeneous Salvo Model

For the purpose of exposition, one calculation of the heterogeneous salvo model for the U.S. force is demonstrated. Specifically, the results at time five are shown. At this point in the battle, the Japanese heavy cruisers attack the *Chicago* and *Canberra* with guns and by torpedoes.

$$\Delta A_6 = \sum_{j=1}^{10}\left(\frac{\varepsilon_{ji}\beta_{ji}\left(\Psi_{ji}B_j\right)}{\varsigma_i}\right)$$

$$\Delta A_6 = \frac{\varepsilon_{1,6}\beta_{1,6}\Psi_{1,6}B_1}{\varsigma_6} + \frac{\varepsilon_{2,6}\beta_{2,6}\Psi_{2,6}B_2}{\varsigma_6} + \ldots + \frac{\varepsilon_{9,6}\beta_{9,6}\Psi_{9,6}B_9}{\varsigma_6} + \frac{\varepsilon_{10,6}\beta_{10,6}\Psi_{10,6}B_{10}}{\varsigma_6}$$

$$\Delta A_6 = \frac{0.090\times2.37\times0.5\times1}{1.655} + \frac{0.090\times2.37\times0.5\times1}{1.655} + \ldots + \frac{0.115\times2.001\times0.5\times17}{1.655} + \frac{0.0\times0.833\times0.0\times0}{1.655}$$

$$\Delta A_6 = 1.504$$

$$A_6 = 0.00$$

The model calculates the amount of offensive power each Japanese ship contributes to the battle. It also includes in this calculation the combat power contributions of the torpedoes fired by the Japanese force. The calculation for the *Canberra* is the same as the *Chicago*. Since no other ships are involved in this event the U.S. force size at the end of time five is 13.

Without showing further computations, we state that the heterogeneous salvo model calculates the loss of the *Vincennes*, *Astoria*, *Quincy*, *Chicago* and *Canberra*. It also calculates that the *Blue* and *Ralph Talbot* are reduced by 56.6%. The Japanese heaver cruisers suffer a 2.3% reduction in strength, while the remaining Japanese ships are reduced by 6.00%. This leads to a continuous combat power loss of 39.64% and pulsed power loss of 11.90% for the U.S. force. The calculated Japanese continuous combat power loss is 1.68% and the pulsed power loss is 2.15%.

3. Conclusions

The combat loss percentages derived from the results of the heterogeneous salvo model's lie within the loss intervals for both continuous and pulsed weapons. The U.S. percentages match those calculated by Beall's simulation, but the values for the Japanese force differ slightly. This difference is a result from the heterogeneous model calculation

of hits against the *Chicago* and *Blue*. Since Beall's data indicated that the Japanese forces targeted group 7, consisting of *Blue* and *Ralph Talbot*, the heterogeneous model assumes a value of 0.5 as input for $\psi_{j,14}$ and $\psi_{j,15}$ ($\forall$ j). The resulting calculations indicate a 56.6% reduction of combat power for both ships, but historically, *Ralph Talbot* was lost and *Blue* suffered no damage, resulting in a 50% combat power loss. If, however, the model concentrates the Japanese combat power at *Ralph Talbot* ($\psi_{j,15} = 1.0$ $\forall$ j), then its results correspond with that of the historical outcome. The resulting continuous power loss remains 39.03% and 10.50% for pulsed power, which is the same as Beall's model. Similarly, the difference in combat power losses for the Japanese force results from the interpretation of the U.S. ψ_{ji}. input parameters.

E. CHAPTER SUMMARY

To utilize the heterogeneous salvo model successfully, it is essential to ensure that the results of its calculations are accurate and meaningful. Beall's naval battle model provides an excellent opportunity to use historical naval battles and their data with which to test and compare the structure of the heterogeneous salvo model. His model, which was based partially on the Hughes Salvo Model, calculated the combat power lost in each force in the battle. Using Beall's data and methodology, this examination has been able to reproduce three representative battles with the heterogeneous salvo equation as the model's core.

By demonstrating that the heterogeneous salvo model is capable of accurately reproducing the results of historical battles, we are able to explore alternate applications of the model. In contrast to the homogeneous salvo model, the heterogeneous model adds an additional layer of depth to salvo warfare, allowing examination of tactics and strategies. Though the homogeneous salvo model might produce acceptable results, using averaged data as model input can lead to optimistic or pessimistic values. The heterogeneous salvo model allows more accurate calculations.

Finally, it has been demonstrated that the heterogeneous salvo model can be used to model continuous fire by partitioning of gunfire into short time steps.

IV. HETEROGENEOUS SALVO MODEL IN NAVAL ANALYSIS

This chapter examines a hypothetical campaign scenario and demonstrate how the heterogeneous salvo model can be used to aid in tactical decision-making. The scenario used in this chapter is based upon an unpublished problem statement [Ref. 14] by Wayne Hughes, Jr., given in his Campaign Analysis course at the Naval Postgraduate School to a group of 11 U.S. and international students. In it he presents a hypothetical maritime conflict between Greece, Turkey, and the United States. The students, divided into groups, were tasked with formulating a campaign strategy for their group and then providing a detailed analysis of a battle.

The concept of operations for the employment of friendly forces within the context of that scenario was developed by Michael Johns, Charles McCaffrey, and Donald Humpart [Ref. 15]. This thesis reflects the application of the heterogeneous salvo model by the author as an analysis tool for the given scenario and concept of operations. This chapter illustrates the differences between the homogeneous and heterogeneous salvo models through comparative analysis of the scenario and concept of operations. The remaining sections demonstrate the heterogeneous salvo model's use in choosing the most desirable weapons load that enables the U.S. forces to successfully accomplish its mission.

A. ANALYSIS OF HYPOTHETICAL U.S. NAVAL ENGAGEMENT

1. Mini-Study Scenario

The purpose of this mini-study scenario is to examine the use of the heterogeneous salvo model to assist in developing, executing, and analyzing a military campaign. Although the circumstances are hypothetical, the scenario has the advantage of using actual order of battles with specific combat characteristics. Thus, it lends itself to clear-cut illustrations of the heterogeneous salvo equations in tactically realistic circumstances. The study examines the United States' ability to prevent the invasion and conquest of five Greek islands in the Aegean Sea by the Republic of Turkey. The two nations had recently engaged in a naval battle over the island of Cyprus in which the Greek military lost half of their ships and aircraft. Using its temporary advantage,

Turkey aspires to reclaim the islands of Límnos, Lésvos, Khíos, Sámos, and Kós, which have been under Greek control for almost a century. Through reliable intelligence sources, the United States finds out about Turkey's plans. The President of the United States orders the U.S. military to prevent the take over. For simplicity of exposition, the Greece force will be regarded as unavailable for direct combat assistance because of their prior losses in the battle for Cyprus.

2. Assumptions

a) United States Forces

The United States Order of Battle consists four DDG-51 Aegis Destroyers and two CG-47 Aegis Cruisers on station in the Mediterranean Sea. Each DDG has 30 SM-2 surface-to-air missiles and eight Harpoon anti-ship cruise missiles, configured in two, quad launchers. Each CG has 40 SM-2 surface-to-air missiles and eight Harpoon anti-ship cruise missiles. The DDGs are capable of defending against nine simultaneous threats, while the CGs can defend against 12 simultaneously. A total of 60 U.S. Air Force F-16, armed with Maverick air-to-air missiles are available, but require basing in theater.

Additionally, the U.S. force includes 40 small, but heavily armed Streetfighter units [Ref. 4]. Since Streetfighter is an evolving concept, with endless possibilities for its design and configuration, this chapter uses a Streetfighter specified in the problem statement [Ref. 14]. Each Streetfighter displaces 500 tons and is capable of sustaining 40 knots and 2000 nautical miles at 20 knots. The Streetfighter is designed to host and operate two Unmanned Aerial Vehicles (UAVs). Their armament includes sixteen Harpoon anti-ship cruise missiles, divided into four quad launchers, positioned such that one-half point in either direction. In addition to passive defensive measures, such as chaff or electronic counter measures, each Streetfighter carries a total of eight Rolling Airframe Missiles (RAM). Once launched, each RAM is capable of independently homing on its intended target and is designed to minimize fratricide. For the analyses that follow, the Streetfighters will fire a maximum of four Harpoon missiles per salvo and up to a maximum of eight RAMs to defend against incoming anti-ship missile salvos. For all combatants, the remaining magazine inventory must be calculated independently after each salvo exchange or until a specific magazine is exhausted.

The Streetfighters are designed to operate in a network-centric fashion, i.e., with a high degree of integration and efficient distribution of fire. The UAVs assist with target detection, identification, tracking, and battle damage assessment (BDA). This gives the U.S. force a scouting and targeting advantage over the Turkish force, allowing them to apply a greater amount of combat potential per salvo.

The U.S. mission is to prevent a Turkish amphibious landing by destroying or turning away the warships, thereby stripping escorts from the amphibious force and increasing their vulnerability to subsequent U.S. attacks. Once the escort forces have been eliminated, it is assumed that the amphibious force will either turn back or be easily destroyed by remaining U.S. forces. The proposed tactic is to divide the force into three task groups, each assigned to an area of operation (AOO). These AOOs are situated off the coast of Turkey, near three naval ports of interest. A task group is responsible for engaging all escorts and afterwards, those amphibious ships that do not return to Turkish ports. For simplicity and uniformity in these analyses, each squadron is composed of twelve Streetfighters and one DDG. The two CGs will operate independently, providing air cover and coordination against Turkish air attack for two of the three AOOs. We will assume that the remaining units are out of theater for repairs and are not considered for further analysis. Each task group has six Streetfighter units and one DDG on station at all time. The other six Streetfighters in each task group cycle from a fixed operating base, e.g. Athens or Thessaloniki, for fuel, supplies, rest, and rearmament, so in effect act as a reserve.

Initially, it is assumed that the offensive and defensive coordination between units is excellent and therefore there is no wasted offensive combat or defensive power. In later analyses, we examine what happens when this assumption is not true. It is also assumed that the defensive systems are intelligent and are capable of engaging threatening targets without duplicative multi-kill. Additionally, the surface units will not attempt to offensively target any Turkish aircraft that enter the AOO. For analytical purposes, the total number of salvo exchanges between the U.S. and Turkish escort force is limited to four.

In this study, the United States does not have an aircraft carrier available. 60 U.S.A.F. F-15s and F-16s, forward deployed in Greek airfields, are similarly divided to cover the three task groups. Thus, 20 are tasked with providing air coverage for each specific AOO. The F-16s operate in pairs, maintaining four aircraft on station by rotating back to base only after being relieved. The aircraft, in coordination with the CGs, try to prevent Turkish aircraft from attacking the U.S. naval force. But, Turkish aircraft that survive an air engagement continue to the AOO where they may engage the U.S. surface group.

b) Turkish Forces

The Turkish escort force is composed of five different groups of ships: 2 ex-U.S. *Gearing Fram I* class Destroyers, 6 ex-U.S. *Perry* class Guided Missile Frigates, 8 *MEKO* 200 Frigates, 8 ex-U.S. *Knox* class Fast Frigates, and 20 fast patrol craft (FPC) of various types. Each of the groups uses the Harpoon anti-ship cruise missile as the primary offensive weapon against enemy ships. All but the *Perry* class have eight total ASCM, distributed in two quad launchers. The *Perry* class ships carry a total of four missiles and use the MK 13 Guided Missile Launching System as the means of delivery. The Turkish Navy has three surface-to-air missiles in inventory. The Sea Sparrow missiles are the primary surface to air missile (SAM) used by all of combatants, except the *Perry* Frigates, whose primary SAM is the SM-1. Since all SAM are relatively equivalent in terms of destructive effect, this study aggregates them all into one surface-to-air missile term.

The primary duty of the Turkish ships is to escort the amphibious ships and embarked personnel from their home bases in Izmir, Gulcuk, and Marmaris to the five islands of interest. It is assumed that the Turkish invasion plans have been predetermined and that once the forces leave port, they proceed directly to the assigned target area. Since the Turkish Navy routinely trains with the United States Navy, it is assumed that they are aware of U.S. tactics and Streetfighter capabilities. Additionally, the Turkish Air Force has a committed 150 attack and fighter aircraft (about 50% of the order of battle) to provide advanced warning of U.S. movements and then attack. It is assumed that all 150 aircraft are capable of attacking the U.S. force with two anti-ship

cruise missiles each. Thus, the Turkish force has a relatively large offensive and defensive potential.

Table 6 summarizes what is termed the standard capabilities for each type of Turkish and U.S. warship. Since the Turkish naval force must perform multiple mission areas (anti-air, anti-surface, and anti-submarine) while escorting the amphibious force, it is assumed that they adopt a conservative, less aggressive battle strategy than the previous attack against the Greek force. A conservative strategy allows the Turkish force to concentrate their ships to protect the amphibious force and minimizes the number of offensive salvos. The U.S. force, enjoying the coordination advantage of a network-centric environment, is capable of employing a less conservative plan in which they aggressively attack the opponent. This coordination advantage, coupled with an aggressive "hit and run" strategy, suggests a significantly larger offensive potential for the Streetfighters.

Group	Total Number of Ships	Number of Harpoon per Salvo	Total Number of Harpoon	Max Number of SAM per salvo	Total Number of SAMs
DD(Fram I)	2	2	8	2	8
FFG (Perry)	6	2	4	2	36
FFG (MEKO)	8	2	8	2	8
FF(Knox)	8	2	4	2	8
FPC	20	1	4	1	8
Streetfighter	40	4	16	8	8
DDG	4	2	16	9	30
CG	2	2	16	12	40

Table 6. Standard Configuration of Missiles and Salvo Size on Platforms

c) *Deterministic Values Used for Salvo Parameters*

Thus far we have described the two models and have examined a descriptive example to assist the readers to familiarize themselves with the salvo models. Mathematically, the combat power, defensive power, and staying power are random variables, whose values are either known with some distribution, or unknown. It is important to provide the model with reasonable approximations for these random variables, either through the use of expert opinion, average value, simulated draw, or expected value function.

In many instances, the expected value of the random variable is easily calculated, or easily known, and is used as a surrogate for the true, unknown value. In both versions of the salvo equations, the offensive and defensive load-out of a ship or force is typically known or can be calculated. The expected value of the actual staying power, the effects of chaff, scouting, anti-scouting, etc. are often used, since they serve as practical estimates. To simplify analysis, the effects of scouting, training, distraction, and defender alertness of both forces are aggregated into two parameters—an offensive effectiveness and defensive effectiveness parameter. In the analysis in Section IV.A., we assume that the Turkish offensive and defensive effectiveness have an expected value of 0.80 and that the U.S. values are 1.0. These values imply that the Turkish Navy is highly trained and effective, but is not perfectly effective, while the U.S. force has perfect offensive and defensive effectiveness.

It is possible for a ship to have different values of staying power in terms of different offensive weapons. For example, a ship with a staying power of two "Penguin" hits may only have a staying power of one "Harpoon" hit. An offensive weapon may have an excessive amount of explosive potential, causing more damage than necessary to place a ship out of action with a single hit. But the number of hits to put a ship out of action is always at least one.

3. Analysis of Battle

To evaluate the potential success or failure of the proposed U.S. tactic, this analysis examines three scenarios for the U.S. force: one squadron vs. the entire Turkish Navy, one squadron vs. 50 percent of the Turkish Navy, and one squadron vs. one-third of the Turkish Navy. With these scenarios we expect to find an upper bound on the size

of the Turkish force that the U.S. can successfully counter, the associated casualties, and the necessary armament. In each case, only those ships of a squadron that are on-station, that is, six Streetfighters and one DDG are examined. Initially, the contributions of aircraft on both sides are omitted, but will be examined in a separate analysis. As in Chapter 3, the analysis uses *Microsoft® Excel* [Ref. 12] Spreadsheet as the calculating platform for the heterogeneous salvo model. Additionally, the analysis uses *Insight.xla Business Analysis Software for Microsoft® Excel* [Ref. 13] simulation plug-in to Excel to assist in generating Monte Carlo simulations.

In the following homogeneous analyses, the value for the parameters α, β, a_3, and b_3 are calculated off-line and then used in then Hughes model. It is in those off-line calculations that the assumption that the Turkish offensive and defensive salvos are only 80% effective are reflected. These calculations also consider that the value of these parameters can never exceed the current level of combat potential (inventory level). To calculate these four parameters the following functions are used:

$$a_3 = \min\left[\frac{\beta B}{A}, \text{A's remaining defensive potential}\right]$$

$$b_3 = \min\left[\frac{\alpha A}{B}, \text{B's remaining defensive potential}\right]$$

$$\alpha = 0.80 \times \min\left[\alpha', \text{A's remaining offensive potential}\right]$$

$$\beta = \min\left[\beta', \text{B's remaining offensive potential}\right]$$

where α' and β' are the maximum offensive salvo sizes.

After each exchange, the remaining offensive and defensive potentials are calculated. For the U.S. and Turkish forces, the remaining offensive potential is calculated as the difference between the offensive potential before the exchange and the number fired in the exchange (β and α respectively). For the U.S. force, the remaining defensive potential is calculated as the difference between the initial defensive potential and the number fired in defense, i.e., to shoot down all of enemy offensive salvos fired. The remaining Turkish defensive potential is calculated similarly except that the total number of missiles fired must be used, including those 20% that were ineffective that

each Turkish unit needed to fire to get the value of a_3. To calculate this value we substitute $(a_3A)0.80$ for a_3A and solve for the theoretical value of a_3^*.

$$\beta B - (a_3 A)0.80 = 0$$
$$a_3^* = \frac{\beta B}{0.80A}$$

a) Homogeneous Analysis: Case I

To establish an upper bound or worst-case, the scenario where a U.S. squadron engages the entire Turkish surface force is first examined. Using the values from Table 6, the aggregate values for each parameter needed in the homogeneous salvo equation are calculated. For example, the aggregated combat potential per ship of the Turkish force is total combat power of the force divided by the number of ships in the force, which equals 1.545. Using these values, in Table 7,the resulting number of units lost for each force is calculated.

Group	Total Number of Ships	Average Offensive Salvo	Average Defensive Salvo	Average Staying Power	Average Offensive Potential	Average Defensive Potential	Offensive/Defensive Effectiveness
Turkish Force	44	1.545	1.545	1.364	4.909	11.818	0.8
U.S. Force	7	3.714	8.143	1.143	14.857	11.143	1.0

Table 7. Aggregated Parameters for Homogeneous Salvo Equations

$$\Delta A = \frac{\beta B - 0.80(a_3 A)}{a_1} = \frac{(7\times 3.714) - 0.80(0.7386\times 44)}{1.364} = 0.00$$
$$\Delta B = \frac{0.80(\alpha A) - b_3 B}{b_1} = \frac{0.80(1.545\times 44) - (7.771\times 7)}{1.143} = 0.00$$

From the calculations it is shown that both forces have enough defensive power to defend against the other's offensive salvo. The outcome will depend on which side exhausts the other side's offensive or defensive potential/missiles carried first.

In order to calculate the results of a second exchange, it is necessary to calculate and aggregate the offensive and defensive potential remaining off-line and then use those numbers as input in the homogeneous salvo equation. In this exchange, the offensive and defensive potential remaining are 3.673 and 11.080 respectively, for the

Turkish force, and 11.143 and 3.371 respectively for the U.S. force. It is clear that the U.S. force expended much more in defense than the Turkish force and, having only 3.371 defensive shots per ship left, will not be able to completely defend against another Turkish offensive.

$$\Delta A = \frac{\beta B - 0.80(a_3 A)}{a_1} = \frac{(7 \times 3.714) - 0.80(0.7386 \times 44)}{1.36} = 0.00$$

$$\Delta B = \frac{0.80(\alpha A) - b_3 B}{b_1} = \frac{0.80(1.545 \times 44) - (3.371 \times 7)}{1.14} = 26.95$$

The homogeneous salvo model suggests that, having six times more ships, the Turkish force is superior to the U.S. force, even when giving the U.S. stronger technological capabilities.

b) Heterogeneous Analysis: Case I

Using the heterogeneous salvo model and the data in Table 6, the calculations indicate a different result. This analysis assumes the Turkish force adopts a 9-to-2 offensive strategy against the Streetfighters and DDG, that is, 9/11 of their combat potential is directed toward the Streetfighters and 2/11 toward the DDG. This tactic represents the minimum allocation of offensive salvos that saturates the DDG's defenses 50 percent of the time. These values were calculated independently by means of Monte Carlo simulation and trial-and-error. Its purpose is not to provide an exact value, but is a good approximation for the Turkish allocation of missiles. The U.S. force also faces a tactical decision—which Turkish group of ships to target and in what order. The U.S. force does not have sufficient capability to concentrate fire and completely destroy any single group of Turkish ships in one salvo except the DDs and FFGs. All others require at least two exchanges. If the decision is to destroy as many enemy ships as possible, then the analysis, shown in Table 8, suggests targeting the DDs and FFG first (because of their weak staying power) then concentrate fire on any one of other groups. In this exchange, the FPCs are targeted in the second exchange, resulting in a total loss of 16 Turkish ships. In any decision, both the Streetfighters and DDG are destroyed by the second engagement. In this example, there are 2.8 hits against the DDs and 10.4 hits against FFGs. These hits are in then distributed equally throughout the entire ship group. In the second exchange, the concentrated fire from the Streetfighters and DDG results in

9.1 hits against the FPCs. Having expended all 48 defensive missiles in the first exchange, the Streetfighters receive 30.55 hits, while the DDG receives 2.57 hits, destroying both groups of ships. Note: In the first exchange, the DDG sustains on average 0.45 hits, which, by the assumption of the Turkish targeting strategy, is what is expected. In any single calculation, there is at least one hit.

Since it is limited to two Harpoon missiles per salvo, the DDG contributes only 5.8 percent (3 of 51) of the total offensive potential of the U.S. force, which is not sufficient to independently destroy even the Turkish group of two DDs. If the Turkish offensive strategy dedicates less combat power toward the DDG, then the destroyer's strong defensive capabilities allows it to successfully counter the Turkish offensive salvo and continue in additional engagements.

	Starting Number	Hits Received (*First Salvo*)	Number Remaining	Hits Received (*Second Salvo*)	Number End
DD(Fram I)	2	2.8	0	0	0
FFG (Perry)	6	10.4	0	0	0
FFG (MEKO)	8	0	8	0	8
FF(Knox)	8	0	8	0	8
FPC	20	0	20	9.1	10.9
Streetfighters	6	0	6	30.55	0
DDG	1	0.45	0.55	2.57	0
CG	0	0	0	0	0

Table 8. Summary of U.S. Forces vs. 100 percent Turkish Force

c) Homogeneous Analysis: Case II

The second case examines a sequence of exchanges between 50 percent of the Turkish force engaged against the on-station U.S. force. The aggregated values, found in Table 7, are used in the homogeneous equation, with only one change—the total number of Turkish ships in this case is 22 instead of 44. Substituting these numbers into the equations, the change in the two forces after the first exchange are as follow:

$$\Delta A = \frac{\beta B - 0.80(a_3 A)}{a_1} = \frac{(7\times 3.714) - 0.80(1.477\times 22)}{1.36} = 0.00$$

$$\Delta B = \frac{0.80(\alpha A) - b_3 B}{b_1} = \frac{0.80(1.545\times 22) - (3.886\times 7)}{1.14} = 0.00$$

Again, the first exchange results in a draw, since each side is capable of defending against the other's offensive salvos. The Turkish offensive (27.2 missiles) is easily defeated by the combined U.S. defensive, which is capable of deploying a maximum of 57.00 missiles (48.858 from the Streetfighters and 8.143 from the DDG). Similarly, the U.S. offense (26 missiles) is defeated by the Turkish force, which is capable of deploying a maximum of 33.99 missiles (27.2 effective). Note that the Turkish force has little room for error without sustaining a hit. As in the first case, the remaining offensive and defensive capabilities are calculated independently and used in the homogeneous salvo equation to calculate the results of the second exchange. In the next exchange, the offensive and defensive potential remaining are 3.364 and 10.341 respectively for the Turkish force, and 11.143 and 7.257 respectively for the U.S. force. Since the U.S. defensive potential is less than 8.143 (the maximum defensive salvo size), its maximum defensive salvo size is now limited to the current inventory level. The Turkish force, on the other hand, has the inventory to sustain their maximum rate of fire. The resulting calculations are as follows:

$$\Delta A = \frac{\beta B - 0.80(a_3 A)}{a_1} = \frac{(7 \times 3.714) - 0.80(1.477 \times 22)}{1.36} = 0.00$$

$$\Delta B = \frac{0.80(\alpha A) - b_3 B}{b_1} = \frac{0.80(1.545 \times 22) - (3.886 \times 7)}{1.14} = 0.00$$

Since both forces are capable of defending against the other's offensive salvos, the second exchange results in a draw. The offensive and defensive potential remaining are 1.818 and 8.864 respectively for the Turkish force, and 7.429 and 3.371 respectively for the U.S. force. The resulting calculations are as follows:

$$\Delta A = \frac{\beta B - 0.80(a_3 A)}{a_1} = \frac{(7 \times 3.714) - 0.80(1.477 \times 22)}{1.36} = 0.00$$

$$\Delta B = \frac{0.80(\alpha A) - b_3 B}{b_1} = \frac{0.80(1.545 \times 22) - (3.371 \times 7)}{1.14} = 3.15$$

After this exchange, the U.S. force is reduced to only 3.85 of 7 ships, while the Turkish Force remains undamaged. The offensive and defensive potential remaining are 0.273 and 7.386 respectively for the Turkish force, and 3.714 and 0.000

respectively for the U.S. force. The U.S. force does not have sufficient defensive potential to counter the Turkish offensive, but the Turkish force has just enough defenses in each exchange to thwart the U.S. attack. In the fourth exchange, the remaining U.S. ships are defeated.

$$\Delta A = \frac{\beta B - 0.80(a_3 A)}{a_1} = \frac{(3.85 \times 3.714) - 0.80(0.813 \times 22)}{1.36} = 0.00$$

$$\Delta B = \frac{0.80(\alpha A) - b_3 B}{b_1} = \frac{0.80(0.273 \times 22) - (0 \times 3.85)}{1.14} = 4.2$$

d) Heterogeneous Analysis: Case II

Using the heterogeneous equation, this analysis assumes the Turkish force adopts a 3-to-2 offensive tactic, that is 3/5 of the offensive is directed towards the Streetfighters and the remainder towards the DDG. It also assumes the Streetfighters offensive tactic is to fire 5% of its offensive power against the DD and 35% against the FFG, and 60% against the Meko. For ease of writing, a particular unit's targeting tactic will be denoted as follows: (0.05, 0.35, 0.60, 0.00, 0.0). In the second exchange, the Streetfighters adopt the following tactic (0.00, 0.00, 0.00, 0.00, 0.95). In the third exchange, the tactic is (0.0, 0.0, 0.0, 0.65, 0.0). The DDG's tactic in the first exchange is (1.0, 0.0, 0.0, 0.0, 0.0). At first it appears that the U.S. force easily destroys the Turkish force, but there are a few items worth noting. First, as in the results from the first heterogeneous analysis, the DDG, contributing 3.1 percent (2 of 64.5) of the total combat power, is relatively ineffective against any Turkish group that has superior numbers. In this instance it is only partially effective against the Turkish destroyer. Second, the only way the DDG is able to survive an exchange is if the Turkish force adopted a strategy where the Streetfighters is the focus of their combat power, i.e., greater than 60 percent combat power, and ignored the DDG. If this happens, the U.S. force would be able to defeat the Turkish force in two exchanges without suffering casualties. Third, if the DDG is omitted from this analysis or if the Turkish force chooses to concentrate fire on the Streetfighters, then the Streetfighters are capable of victory without suffering casualties.

	Starting Number	Hits Received *(First Salvo)*	Number Remaining	Hits Received *(Second Salvo)*	Number Remaining	Hits Received *(Third Salvo)*	Number End
DD(Fram I)	1	1.58	0	0	0	0	0
FFG (Perry)	3	3.6	0	0	0	0	0
FFG (MEKO)	4	8.02	0	0	0	0	0
FF(Knox)	4	0	4	0	4	9.2	0
FPC	10	0	10	13.6	0	0	0
Streetfighters	6	0	6	0	6	0	0
DDG	1	1.88	0	0	0	0	0
CG	0	0	0	0	0	0	0

Table 9. Summary of U.S. Force vs. 50 percent Turkish Force

e) Homogeneous Analysis: Case III

This case examines a sequence of exchanges between the U.S. force that is on-station and one-third of the Turkish Force. Again, the initial values from Table 7 are used as input for the first exchange in the homogeneous model. The results of the first salvo is as follows:

$$\Delta A = \frac{\beta B - 0.80(a_3 A)}{a_1} = \frac{(7\times 3.714)-0.80(1.545\times 14.67)}{1.364} = 5.769$$

$$\Delta B = \frac{0.80(\alpha A) - b_3 B}{b_1} = \frac{0.80(1.545\times 14.67)-(2.590\times 7)}{1.14} = 0.00$$

The second salvo exchange is given by:

$$\Delta A = \frac{\beta B - 0.80(a_3 A)}{a_1} = \frac{(7\times 3.714)-0.80(1.545\times 8.89)}{1.364} = 10.99$$

$$\Delta B = \frac{0.80(\alpha A) - b_3 B}{b_1} = \frac{0.80(1.545\times 8.89)-(1.572\times 7)}{1.14} = 0.00$$

This analysis indicates that the strength and capability of the U.S. force is sufficient to overwhelm the Turkish defenses and score enough hits to weaken the force in the first salvo exchange, and destroy it in the second exchange.

f) Heterogeneous Analysis: Case III

The heterogeneous case is trivial, since the U.S. force was shown to be capable of achieving its objective in the previous case. There are two points worth noting, however. First, even at this minimal number of enemy force, the DDG is still ineffective against all enemy groups except the DD. Second, the Turkish offensive

strategy that defines the threshold for the DDG is almost 1-to-1. This implies that the Turkish force would have to allocate approximately 50 percent (9 of 18 salvos) of their available combat potential just to successfully destroy the DDG. Since the group of Streetfighter, with a defensive salvo capability of 24 in the first salvo, can easily defend against the entire Turkish combat potential of 18 missiles. This analysis suggests that the best strategy for the Turkish force is to concentrate enough of their first offensive salvo to eliminate the DDG, because defensively it is limited to nine SAMs.

g) Turkish Air Threat

For clarity in the analysis of the first three cases, the effects of Turkish aircraft were ignored. Since the air strike analysis is not needed to show how the details of the heterogeneous equations are used, its effects are merely summarized here. In Case I, the U.S. force is not capable of surviving the first salvo exchange and so a strike by the aircraft is redundant. Case II represents the threshold at which the Streetfighters can defend against the Turkish offensives and continue to the next exchange. A sufficiently large strike by the aircraft weakens the U.S. force so that it is saturated by the Turkish surface force's attack and is destroyed in the first exchange. Only in Case III is there sufficient slack in the U.S. force's defense to evaluate the maximum number of aircraft that can it can face and still succeed in the mission. From the previous analysis, in the best Turkish scenario, the Turkish force has a maximum combat power of 24 missiles, while the U.S. force has a defensive power of 33 missiles. This suggests that the U.S. can engage an additional nine missiles, or 4.5 aircraft, before saturation.

In all cases, if the aircraft engage the Streetfighters in sufficient numbers before the first exchange of salvos between surface combatants, then the U.S. force is overwhelmed by the aircraft and destroyed. U.S. Air Force aircraft cannot attack with ASMs but are only useful as CAP to cover the U.S. ships and disrupts the Turkish air strikes.

4. Summary

This analysis illustrates the differences between the homogeneous and heterogeneous salvo models. Where the homogeneous model provides quick and simple insight in to the characteristics of salvo warfare, it fails to account for the unique

characteristics of platforms and the usefulness of these characteristics. The heterogeneous model is capable of capturing these essential details while maintaining the relative simplicity of the original salvo model. The heterogeneous model, however, entails performance inputs and tactical details, which can quickly grow and must be tracked by the modeler.

As to substance of the results, the illustrative study shows that the U.S. force is capable of meeting the campaign objective. Each task group is able to engage up to 50 percent of the Turkish naval force and still meet the objectives. However, as the Turkish force size increases, so too does the number of U.S. casualties suffered. It has been shown that the DDG does not contribute enough combat power to dramatically alter the numerical results and, depending on the Turkish strategy, is vulnerable to a large Turkish salvo. This result, coupled with the group of Streetfighter's ability to successfully engage the Turkish force without the DDG, suggests an alternate plan. Rather than exposing the DDG to unnecessary surface engagements, the DDG should remain clear of battle and defend against Turkish air attacks.

B. STOCHASTIC ANALYSIS USING THE HETEROGENEOUS SALVO MODEL

The simplicity of the salvo equation is both a major benefit as well as a drawback. Though the equations may provide quick and easy results, oversimplifying the assumptions limits the insights gained. The usefulness of the heterogeneous salvo model, by contrast, depends on the targeting data provided for each platform, and also upon the parameters used to describe the platform's effectiveness. We next wish to show how the heterogeneous salvo model can be used with stochastic variation of parameters by assigning probabilities to key inputs. We desire to capture the essence of the U.S. and Turkish force interactions and are less concerned with specific or exact values for offensive and defensive effectiveness. To do so requires useful approximations of the true values for these two forces. In the absence of empirical data, a distribution from which to draw a set of random values to use in the model must be assumed.

The triangle distribution is a rough but practical surrogate for the true, but unknown, distribution of both the offensive and defensive effectiveness parameters in the

heterogeneous salvo model. Identifying a feasible interval [a, b], within which it is reasonable to believe the value of the true effectiveness parameter lies (with a < b), establishes necessary criteria to use this distribution [Ref. 16]. An additional value, c, which is the mode of the distribution or its most likely value, is required to give shape to the distribution. The values of a, b, and c are subjective and must be based upon tests, analysis, intelligence, or expert opinion. However, these subjective minimum and maximum values are absolute limits and do not allow values outside this range and in certain circumstances lead to problems. There are two ways to proceed. First, the triangle distribution can use the 0.05 and 0.95 percentiles instead of minimum and maximum values. Second, since the triangle distribution is a special form of the beta distribution, this general distribution, with the appropriate shape parameters, is used. [Ref. 16] Since the salvo equation is based upon many simplifying assumptions, it is not necessary to add additional complexity to the problem in order to capture the exact distribution. Therefore, this study will use the distribution without concern for the minimum and maximum bounds.

Throughout Sections B.1 and B.2 of this chapter, the Turkish offensive and defensive effectiveness are assumed distributed triangular with minimum value of 0.5, maximum of 0.9, and mode of 0.8. On average, an approximate degradation of 20 percent in the offensive and defensive power of the Turkish force is expected. However, the range of degradation can be between 10 and 50 percent. These values imply that the Turkish Navy is highly trained and effective force, but does not have perfect effectiveness.

1. Six Streetfighters vs. 50 Percent Opposition Force

a) Analysis Baseline and Purpose

The previous analysis (Section IV.A.3) demonstrated the heterogeneous salvo model's use in evaluating a potential U.S. tactic and gain insight into an alternative tactical approach. Since the DDG is vulnerable to increased levels of Turkish salvos and since it does not significantly contribute to the total U.S. combat potential, it should not be used to attack the Turkish escorts. If Streetfighters are considered an expendable asset, that is, their loss is preferred to the loss of larger warships, then it is desired to equip the Streetfighter with the least combat potential necessary to achieve victory in an

assigned task. The purpose of this analysis is to demonstrate the use of the heterogeneous salvo model in a Monte Carlo simulation to evaluate the minimum offensive and defensive salvo sizes necessary for one squadron of Streetfighters to successfully engage and defeat the Turkish Navy in a surface naval battle. In this way one can infer whether a different, smaller capability may be better. A smaller Streetfighter design might then allow procurement of a larger Streetfighter force for the same cost.

In the previous analysis, the analysis indicates that if U.S. intelligence is incorrect and the Turkish send more than 50 percent of their naval force against a U.S. task group, then the U.S. would fail in its objective. As before, the 36 operational Streetfighters are divided into three task groups, with six ships on station in each task group. Table 10 provides a summary of a sequence of average exchanges between six Streetfighters, alone, facing fifty percent of the Turkish escort force (22 ships). This results will serve as a baseline from which to compare later results. They are similar to the results in Section IV.A.3.d, but differ in detail because the DDG is removed and because the distribution of is concentrated against the Streetfighers.

	Starting Number	Hits Received *(First Salvo)*	Number Remaining	Hits Received *(Second Salvo)*	Number Remaining	Hits Received *(Third Salvo)*	Number End
DD(Fram I)	1	0.930	0.07	2.219	0	0	0
FFG (Perry)	3	2.802	0.198	1.857	0	0	0
FFG (MEKO)	4	4.268	0	1.171	0	0	0
FF(Knox)	4	0	4	5.461	0	0	0
FPC	10	0	10	0	0	16.649	0
Streetfighters	6	0	6	0.005	5.998	1.5147	4.483
DDG	0	0	0	0	0	0	0
CG	0	0	0	0	0	0	0

Table 10. Summary of Average Exchanges and Results

With tactically advantageous targeting (though not mathematically optimal) of the five Turkish groups, the six Streetfighters are able to significantly reduce the DD, *Perry*, and *MEKO* in the first salvo by expending 25 percent (24 of 96) of their offensive potential and 52 percent (25 of 48) of their SAM capability. In the second salvo, the *Knox* FF is the primary target of the Streetfighters, with 70 percent of the combat potential being targeted at the FF. The remaining 30 percent is distributed equally among the surviving DD and the two FFGs. The FPCs are destroyed in the third

salvo after shooting an additional 24 Harpoon missiles and approximately 7 SAMs. On average, though, the Streetfighters receive 1.5147 hits in the third exchange. This results when the Turkish force has great success in targeting and defense early, causing the Streetfighters to expend many SAMs early in the battle. If the Streetfighters were capable of adjusting their mix of weapons loaded prior to leaving port, it would be beneficial to know how sensitive this result is to changes in the offensive and defensive parameters.

b) Monte Carlo Simulations

By varying the number of Harpoon shots per salvo, one can gain some insight into the Harpoon salvo required, with respect to the SAM salvo size, to achieve the stated MOE and the number of casualties sustained by the Streetfighters. Table 11 is a summary of 1000 Monte Carlo simulations for a fixed SAM salvo size of zero and Harpoon salvo sizes varying from five to nine. In other words, the Harpoons must destroy the entire Turkish force in the first exchange. The results of the simulation show that for salvo sizes less than eight, the average number of Turkish ships that survive the battle is greater than one. The reason for such a large offensive salvo is that, as mentioned, the Streetfighters do not have sufficient defensive capabilities to counter the large Turkish offensive salvo in the first exchange and are, therefore, destroyed before expending all their combat potential. It is claimed that if the Streetfighters had no defensive capability, then they require an offensive capability of at least eight Harpoons per salvo, that is, a lower bound on Harpoon salvo sizes is eight for a SAM salvo size of zero. But mission success is tempered by the loss of all six Streetfighters.

Harpoons/Salvo		9	8	7	6	5
Num Turkish Ships Remaining	**Average**	0.000	0.758	3.894	8.116	13.079
	Std Dev	0.003	0.732	1.517	1.844	2.053
	Std Err	0.000	0.023	0.048	0.058	0.065
	Max	0.078	2.960	6.860	11.946	17.046
	Min	0.000	0.000	0.385	3.752	7.703

Table 11. Simulation with SAM Fixed at Zero.

The opposite extreme was examined using a simulation with the number of SAM fixed at eight and a variable Harpoon salvo size. Table 12 is a summary of the Monte Carlo simulation for Harpoon salvo sizes from two to six. On average, the Turkish escorts survive an engagement with the Streetfighters for offensive salvo sizes less than three. The smaller the offensive salvo sizes, the more exchanges are necessary to expend sufficient combat power on the enemy force. However, the Streetfighters only have a finite supply of defensive weapons and so the Harpoon salvo size must be large enough to destroy the enemy before the inventory of SAMs runs out. With an offensive salvo size of four (from Table 12) and using tactically optimal targeting, the Streetfighters are able to engage and defeat the Turkish ships in three salvo exchanges. Not shown in Table 12, separate calculations indicate that the Streetfighter force suffers an average of 1.789 hits in the third exchange, resulting in the loss of that same number of ships.

Harpoons/Salvo		6	5	4	3	2
Num Turkish Ships Remaining	**Average**	0.000	0.000	0.003	4.420	17.809
	Std Dev	0.000	0.000	0.079	3.319	2.878
	Std Err	0.000	0.000	0.003	0.105	0.091
	Max	0.000	0.000	2.513	16.564	21.391
	Min	0.000	0.000	0.000	0.064	10.020

Table 12. Simulation with SAM Fixed at Eight

From these two simulations we get results for two extreme points and can say that the lower bound on Harpoon salvo size is between four and eight. To try to decrease this gap, another simulation was run and the data analyzed. Table 13 is a summary of the results from a simulation where the SAM salvo size is fixed at four and the Harpoon salvo size again was varied from five to eight.

Harpoons/Salvo		9	8	7	6
Num Turkish Ships Remaining	Average	2.322	2.502	2.765	3.117
	Std Dev	3.410	3.600	3.856	4.169
	Std Err	0.108	0.114	0.122	0.132
	Max	11.000	11.000	11.000	11.000
	Min	0.000	0.000	0.000	0.000

Table 13. Simulation with SAM Fixed at Four

It is observed that for all intermediate values, the Streetfighters are incapable of successfully completing the objective. These configurations appear to posses the unfavorable characteristics of the first two cases. First, the Streetfighters are unable to counter the initial Turkish offensive because they lack sufficient defenses and are destroyed before delivering enough salvos against the Turkish force. Second, the Streetfighters do not have enough defensive capability to survive a Turkish salvo. Taking a risk tolerant targeting strategy, it is possible to reduce the total Turkish force to less than one ship with a Harpoon salvo size of six and a SAM size of four 64 percent of the time with an average of 3.60 Streetfighters remaining. However, 36 percent of the time the average number of Turkish ships surviving is 5.367 and zero Streetfighters remaining. Table 14 shows that for a little larger SAM salvo of five, the Streetfighters need a Harpoon salvo size of five to complete the mission.

Harpoons/Salvo		5	4	3	2
Num Turkish Ships Remaining	Average	0.005	0.010	4.912	16.292
	Std Dev	0.123	0.193	1.953	2.296
	Std Err	0.004	0.006	0.062	0.073
	Max	3.829	5.502	12.071	20.760
	Min	0.000	0.000	0.685	10.103

Table 14. Simulation with SAM Fixed at Five

c) Summary

The large size of the Turkish escort force (22 ships vs. six Streetfighters) dictates the need for the Streetfighter's configuration to either allow a sufficiently large Harpoon salvo size or large defense to achieve victory. By examining Figure 3, the minimum Harpoon salvo size required to destroy the Turkish force given a known SAM

salvo size is determined. The opposite is also easily determined. Given a fixed Harpoon capability, the minimum SAM capabilities needed to accomplish the mission is determined. The analysis also suggests a risk tolerant strategy, resulting in large payoffs, is possible given an optimal targeting strategy.

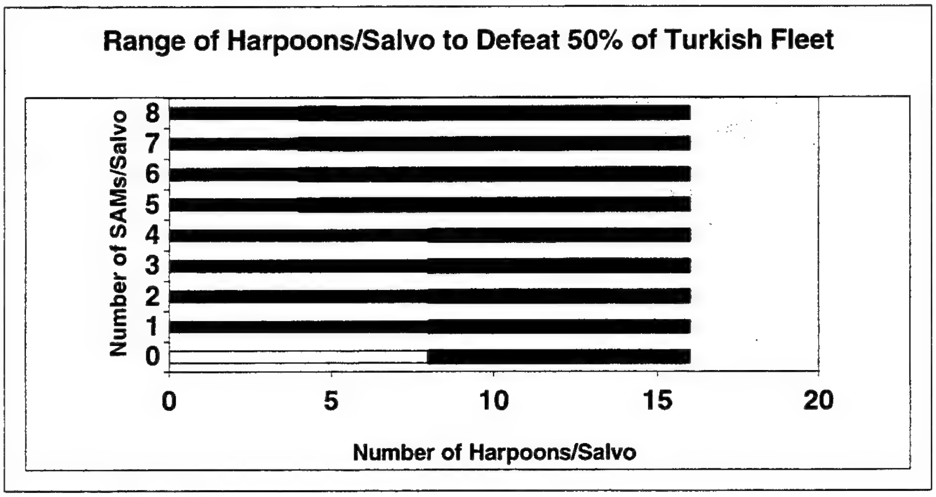

Figure 3. Required Harpoon-to- SAM Salvo Sizes Against 50% Opposing Fleet

2. Six Streetfighters vs. 33 Percent Opposition Force

a) Baseline Analysis

The U.S. Concept of Operations stated that the Turkish invasion force is equally divided in three naval regions along the western coast of Turkey. It also states that the Streetfighters task group will immediately engage the invasion forces after they leave port, preventing them from joining together to form one large force. Table 15 is a summary of the exchanges between one-third of the Turkish Navy and the Streetfighters with standard configuration (four Harpoon/salvo and eight SAM/salvo). Selecting an appropriate targeting plan, the Streetfighters are easily capable of defeating three of the five Turk groups, expending 21 Harpoons and 17 SAMs in the first salvo. All six Streetfighters survive the second salvo, expending 24 Harpoons and eight SAMs against the last group.

	Starting Number	Hits Received (*First Salvo*)	Number Remaining	Hits Received (*Second Salvo*)	Number End
DD(Fram I)	.667	1.42	0	0	0
FFG (Perry)	2	3.05	0	0	0
FFG (MEKO)	2.667	0	2.667	8.07	0
FF(Knox)	2.667	0	2.667	8.07	0
FPC	6.667	8.29	0	0	0
Streetfighters	6	0	6	0	6

Table 15. Summary of Exchanges Against 1/3 of Turkish Forces

b) Monte Carlo Simulation

Again, the objective is to determine the minimum salvo capabilities necessary for the Streetfighters to successfully complete the mission objective. The first simulation fixes the SAM salvo size at zero missiles and varies the Harpoon salvo size. Since there are no defenses, the Streetfighters must have sufficient combat potential to destroy the Turkish force in the first exchange. From Table 16, the Streetfighters require a minimum Harpoon salvo size of seven in order to meet the U.S. objectives.

Harpoons/salvo		7	6	5	4	3
Num Turkish Ships Remaining	**Average**	0.001	0.648	3.978	8.586	12.619
	Std Dev	0.010	0.575	1.449	1.413	1.066
	Std Err	0.000	0.018	0.046	0.045	0.034
	Max	0.131	2.148	6.828	11.037	14.352
	Min	0.000	0.000	0.679	4.760	9.440

Table 16. Simulation with SAM Fixed at Zero

From the baseline calculation, for an available SAM salvo size of eight, the Streetfighters need at most a Harpoon salvo capability of four to defeat the Turkish force. Further analysis shows that the offensive salvo size can be as small as two missiles when the Streetfighters use a (0.15, 0.0, 0.85, 0.0, 0.0), (0.0, 0.0, 0.0, 0.00, 1.0), (0.0, 0.0, 0.0, 1.0, 0.0), (0.0, 1.0, 0.0, 0.0, 0.0) strategy in the first, second, third, and fourth engagements, respectively. It isconcluded that, for this battle, the required Harpoon salvo size is between two and seven missiles per salvo.

	Starting Number	Hits Received *(First Salvo)*	Number Remaining	Hits Received *(Second Salvo)*	Number Remaining	Hits Received *(Third Salvo)*	Number Remaining	Hits Received *(Fourth Salvo)*	Number End
DD	.667	0.775	0	0	0	0	0	0	0
Perry	2	0	2	0	2	0	0	0	0
MEKO	2.667	6.98	0	0	0	0	0	9.473	0
FF	2.667	0	2.667	0	2.667	3.949	0	0	0
FPC	6.667	0	6.667	6.873	0	0	4.483	0	0
Streetfighters	6	0	6	0	6	0	9	0	6

Table 17. Summary of Exchanges Against 1/3 of Turkish Force And SAM Salvo Size of Three Missiles

For a SAM salvo size of three, the Streetfighters are capable of defending themselves through three salvos and completely destroying the Turkish force by the fourth exchange with a Harpoon salvo size of two. For SAM sizes less than three, the corresponding Harpoon size increases dramatically.

c) Summary

From the simulations, it is concluded that to minimize the Harpoon salvo size while still achieving the mission objective requires a defensive salvo size capable of allowing the Streetfighters to remain in the engagement. If the U.S. force battles 33 percent of the Turkish force, then the minimum configuration necessary is a defensive capability of three or more per salvo and an offensive salvo size of two. Figure 4 graphical depicts the range of Harpoons salvo sizes necessary to achieve victory given a fixed SAM salvo size. The figure indicates that there is less flexibility in the Streetfighter's offensive salvo size compared to that of the defensive salvo size. If there is a need to minimize capabilities, say for cost reasons, then reducing the defensive salvo size up to the indicated levels will satisfy the mission objective at a risk of increased losses to the Streetfighters.

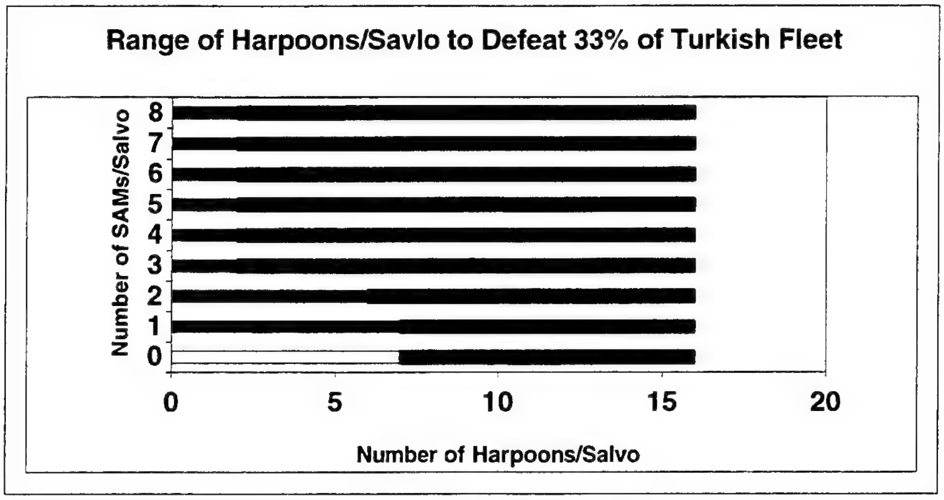

Figure 4. Required Harpoons-to-SAM Salvo Sizes Against 33% Opposing Fleet

C. CHAPTER SUMMARY

1. Heterogeneous Salvo Model

From the three scenarios, the analyses demonstrate the use of the heterogeneous salvo model in support of decision-making. The homogeneous salvo model provides quick insight into the general composition of force design and can be used to assist with tactical decisions when forces are relatively equally capable. The U.S. Navy's vision of the future combat environment suggests that it will face threats of various types, with different capabilities. The heterogeneous salvo model captures these characteristics to calculate the results of a salvo exchange. The three scenarios presented here demonstrate the usefulness of the model and illuminates the differences between the homogeneous and heterogeneous salvo models.

2. U.S. Force Design and Weapons Mix

In this analysis the possible deployment of U.S. forces and the tactical use of those forces are examined. Approaching the problem from a risk-averse viewpoint, the scenarios examine the problem by exploring the threshold of mission failure. With full postulated Streetfighter capabilities, the examination indicated that the engaged U.S. force (six Streetfighters) is capable of meeting the mission objectives when it engages a maximum of 50 percent of the Turkish Force. In all scenarios, only a fraction of the U.S. force engages in combat; the remainder serve as reserves and could be used tactically to augment the force where needed. The analysis shows that when larger U.S. warships

engage in combat, they are easily overwhelmed and destroyed when the enemy has superior numbers. In this situation, these ships would serve better in their primary mission areas, providing air and submarine support for the Streetfighters. By examining the minimum required weapons load that ensures mission success, the analysis demonstrates the flexibility of the heterogeneous salvo model to explore tactical decisions.

THIS PAGE INTENTIONALLY LEFT BLANK

V. CONCLUSIONS

Joint Vision 2020 and *Forward...From the Sea, The Navy Operational Concept* offer Navy leaders insight into where and how potential military threats will manifest themselves and how U.S. military forces must work collaboratively ("jointly") to fight against these threats. As technological advances emerge, the U.S. Navy must seize those capabilities that will continue to propel its forces ahead of those of potential adversaries. These advances cannot be limited to technologies, but must be a part of a system of innovations. Such systems include, but are not limited to, doctrine, tactics, logistics, and training. Military and civilian leaders will constantly require analysis of these new capabilities, supported by analytical and simulation models. With the emergence of these advances, analysts will require either newly developed models to test these advances, or old models capable of keeping up with the changes. This thesis has demonstrated that in the arena of salvo warfare, the heterogeneous salvo model is flexible enough to give the analysts broad insight into the characteristics of the naval forces.

A. AN ANALYTICAL MODEL AND ITS APPLICATION FOR THE NAVY AFTER NEXT

The Navy Warfare Development Command and the Strategic Studies Group are leading the U.S. Navy's efforts to chart its path. The Capabilities for the Navy After Next (CNAN) project is an attempt by the U.S. Navy to examine those leading technologies, capabilities, and doctrine that best contribute to future naval forces. A concept under examination is network-centric operations (NCO), which shifts operational focus from platform to the network in which they operate. The Navy After Next will consist of multiple types of platforms, sensors, and weapons, networked together to operate efficiently, effectively and adaptively. The power of these network-centric forces comes from the timely exchange of information between units within the network.

If the Navy After Next is to be a force founded upon the principles of network-centric operations, then the U.S. Navy will need to pursue innovative methods to coordinate, operate and sustain joint forces. Analysts will be required to examine future systems, problems, or operational methods. These analysts will need methodologies and

models that are flexible enough to keep up with the dynamically changing warfare environment.

The heterogeneous salvo model, an extension of the Hughes Salvo Model, is a flexible and robust analytical combat model to show networking advantages. The model's strength lies in its ability to mathematically capture the interaction of a heterogeneous force, engaged in salvo warfare against an opposing force, at the individual unit level. By encapsulating the unique characteristics of all units, the heterogeneous salvo model gives the analyst the capability to exploit unique combat characteristics and determine their potential contributions to the force. The model's potential as a tactical or planning tool was demonstrated in the analysis of three hypothetical scenarios. Unlike the alternatives, the heterogeneous salvo model allows broad yet simple analysis to improve the tactical distribution of combat power. It allows a force's combat potential to be allocated advantageously among many targets for any salvo.

Though developed to analyze salvo attrition, this thesis has also demonstrated that the heterogeneous salvo model is flexible enough to model continuous force weaponry, such as gun systems.

There is a cost for using the heterogeneous salvo model. Unlike the homogeneous salvo model, which has only a few parameters, the number of parameters used by the heterogeneous salvo model grows as the number of unit types increases. For relatively small force sizes, the model's calculations can be done by hand or calculator. Spreadsheet analysis, as done in this thesis, can implement moderate sized forces, but require a large amount of setup time. This setup time and the subsequent calculations may prevent the model from being useful for immediate time critical tactical solutions.

The heterogeneous salvo model is a combat model with endless applications in the Navy After Next. It is an analytical tool to help decision makers and their staffs easily understand the complex relationships that exist within and between forces during battle. Yet it is easy to understand, program, and apply, given access to inputs.

B. RECOMMENDATIONS FOR FURTHER RESEARCH

While not all-inclusive, the following list details some recommendations for further study to help develop the heterogeneous salvo model for use by naval tacticians:

- The use of graphical user interfaces to support the application of the heterogeneous salvo model in high-level computer languages.

- Conduct an analysis of land and air based aircraft units engaging naval forces. Compare the tactical use of aircraft currently in inventory against planned inventories in support of naval operations.

- Examine the tactical use of "smart" or "next generation" of stand-alone weapons in naval tactics. Analyze the introduction of mines, sensors, or other devices, in a tactical network of naval platforms as independent units.

THIS PAGE INTENTIONALLY LEFT BLANK

LIST OF REFERENCES

1. Chairman of the Joint Chiefs of Staff, "Joint Vision 2020," June 2000.

2. Secretary of the Navy, "Forward...From the Sea, The Navy Operational Concept," March 1997.

3. Cebrowski, A.K. and J.J. Garstka, "Network-Centric Warfare, Its Origin and Future," Naval Institute Proceedings, January 1998.

4. Cebrowski, A.K. and W.P. Hughes, "Rebalancing the Fleet," Naval Institute Proceedings, November 1999.

5. Hughes, W.P., Jr., *Fleet Tactics: Theory and Practice*, Naval Institute Press, Annapolis, MD, 1986.

6. Stein, J., *The Random House College Dictionary Revised Edition*, Random House, Inc., 1980.

7. Hughes, W.P., Jr., "A Salvo Model of Warships in Missile Combat Used to Evaluate Their Staying Power," in *Warfare Modeling*, Bracken, Kress, Rosenthal (Eds.), Wiley, pp, 121-143, 1995.

8. Hughes, W. P., Jr., "Take the Small Boat Threat Seriously," Naval Institute Proceedings, October 2000.

9. Sakellariou, D., "The Effect of Staying Power on Offensive and Defensive Power of A Modern Warship," Masters Thesis, U.S. Naval Postgraduate School, Monterey, CA, 1993.

10. Operations Research Department, "Aggregate Combat Models," Naval Postgraduate School, Monterey, CA, 2000.

11. Beall, T.R., "The Development of a Naval Battle Model and Its Validation Using Historical Data," Masters Thesis, Naval Postgraduate School, Monterey, CA, 1990.

12. Microsoft Corporation, *Microsoft® Excel 2000*, Version 9.0.2720, 1985-1999.

13. Savage, S.L., *Insight.xla Business Analysis Software for Microsoft® Excel*, Version 1.1, Brooks/Cole Publishing Company, 1998.

14. Hughes, W.P., Jr., "Ministudy Scenario, Summer 2000," paper presented for OA4602 Joint Campaign Analysis, Naval Post Graduate School, Monterey, CA, July 2000.

15. Humpart, D., McCaffrey, C.M., and Johns, M.D., "Ministudy Scenario: Concepts of Operation and Analysis for U.S. Team II," Naval Postgraduate School, Monterey, CA, September 2000.

16. Law, A. M. and Kelton, W. D., *Simulation Modeling & Analysis*, McGraw-Hill, Inc., 1991.

BIBLIOGRAPHY

1. Cares, J. R., "The Fundamentals of Salvo Warfare," Masters Thesis, Naval Postgraduate School, Monterey, CA, 1990.

2. McGunnigle, J., Jr., "An Exploratory Analysis of the Military Value of Information and Force," Masters Thesis, Naval Postgraduate School, Monterey, CA ,1999.

3. Smith, T. T., "Combat Modeling Low Intensity Conflict Anti-Surface Warfare for Engagement Analysis(U)," Masters Thesis, Naval Postgraduate School, Monterey, CA, 1991.

THIS PAGE INTENTIONALLY LEFT BLANK

INITIAL DISTRIBUTION LIST

1. Defense Technical Information Center 2
   8725 John J. Kingman Road, Suite 0944
   Ft. Belvoir, VA 22060-6218

2. Dudley Knox Library 2
   Naval Postgraduate School
   411 Dyer Road
   Monterey, CA 93943-51013

3. Research Office Code 09 1
   Naval Postgraduate School
   Monterey, CA 93943-5138

4. Institute for Joint Warfare Analysis Code JW 5
   Naval Postgraduate School
   Monterey, CA 93943

5. VADM Arthur K. Cebrowski 1
   President
   Naval War College
   686 Cushing Road
   Newport, RI 02841

6. RADM Robert G. Sprigg 1
   Commander
   Navy Warfare Development Command
   686 Cushing Road
   Newport, RI 02841

7. Chief of Staff 1
   Navy Warfare Development Command
   686 Cushing Road
   Newport, RI 02841

8. Technical Director 1
   Navy Warfare Development Command
   686 Cushing Road
   Newport, RI 02841

9. Director 1
Maritime Battle Center
Navy Warfare Development Command
686 Cushing Road
Newport, RI 02841

10. Department Head 1
Operations Department
Navy Warfare Development Command
686 Cushing Road
Newport, RI 02841

11. Department Head 1
Concepts Department
Navy Warfare Development Command
686 Cushing Road
Newport, RI 02841

12. Department Head 1
Doctrine Department
Navy Warfare Development Command
686 Cushing Road
Newport, RI 02841

13. Deputy Department Head 1
Concepts Department
Navy Warfare Development Command
686 Cushing Road
Newport, RI 02841

14. Deputy Department Head 1
Operations Department
Navy Warfare Development Command
686 Cushing Road
Newport, RI 02841

15. Deputy Director 1
Maritime Battle Center
Navy Warfare Development Command
686 Cushing Road
Newport, RI 02841

16. Deputy Department Head 1
Doctrine Department
Navy Warfare Development Command
686 Cushing Road
Newport, RI 02841

17. Mr. Hal Hultgren...1
Naval Undersea Warfare Center Code 601
Newport, RI 02841

18. Mr. Frank White..1
Space and Naval Warfare Systems Center Code D11
53560 Hull Street
San Diego, CA 92152-5001

19. Mr. Ray E. Glass 1
Space and Naval Warfare Systems Center Code D4402
53560 Hull Street
San Diego, CA 92152-5001

20. Dr. Richard Kass 1
Analysis Division Chief
U.S. Joint Forces Command
Joint Experimentation, J97
1562 Mitscher Ave. Suite 200
Norfolk, VA 23551-2488

21. Ms. Annette Ratzentberger 1
U.S. Joint Forces Command Joint Experimentation, J95
1562 Mitscher Ave. Suite 200
Norfolk, VA 23551-2488

22. Mr. Vince Roske, Jr. 1
The Joint Staff, J8
The Pentagon
Washington, DC 20318-8000

23. Center for Naval Analyses..1
4401 Ford Avenue
Alexandria, VA 22302-0268

24. Dr. John Hanley 1
Commander-in-Chief USCINCPAC/J00
Box 64031
Camp H M Smith, HI 96861-4031

25. Dr. Moshe Kress 1
CEMA
P.O.B. 2250 (TI)
Haifa, ISRAEL 31021

26. Mr. Andrew Marshall 1
Director of Net Assessment Office of the Secretary of Defense
The Pentagon, Room 3A930
Washington, DC 20301

27. Prof. Gordon Schacher 5
Institute for Joint Warfare Analysis
Naval Postgraduate School
Monterey, CA 93943-5101

28. Prof. William Kemple 1
Institute for Joint Warfare Analysis
Naval Postgraduate School
Monterey, CA 93943-5101

29. Prof. Walter LaBerge 1
Institute for Joint Warfare Analysis
Naval Postgraduate School
Monterey, CA 93943-5101

30. Prof. Phil Depoy 1
Institute for Joint Warfare Analysis
Naval Postgraduate School
Monterey, CA 93943-5101

31. Prof. Steven E. Pilnick 5
Institute for Joint Warfare Analysis
Naval Postgraduate School
Monterey, CA 93943-5101

32. Prof. Wayne P. Hughes, Jr. 5
Dept of Operations Research
Naval Postgraduate School
Monterey, CA 93943-5101

33. Prof. Shelley Gallup 1
Institute for Joint Warfare Analysis
Naval Postgraduate School
Monterey, CA 93943-5101

34. Prof. Alex Callahan..1
Institute for Joint Warfare Analysis
Naval Postgraduate School
Monterey, CA 93943-5101

35. Mr. Curtis Blais..1
Institute for Joint Warfare Analysis
Naval Postgraduate School
Monterey, CA 93943-5101

36. Jeffrey R. Cares..1
31 Willow Street
Newport, RI 02840

37. Michael Johns 3
209 Worden Street
Portsmouth, RI 02871

38. Robert L. Johns 1
13951 SW 106 Street
Miami, FL 33186-3130

Naval Postgraduate School
Monterey, California